Atmospheric Refraction at Optical Wavelengths:

Problems and Solutions

V. B. Mendes[(1)] and E. C. Pavlis[(2)]

[(1)]Laboratório de Tectonofísica e Tectónica Experimental and Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa, Lisboa, Portugal (vmendes@fc.ul.pt)
[(2)]Joint Centre for Earth Systems Technology, University of Maryland Baltimore County, NASA Goddard 926, Greenbelt, MD 20771, USA (epavlis@JCET.umbc.edu)

Abstract

Atmospheric refraction is an important accuracy-limiting factor in the application of satellite laser ranging (SLR) to high-accuracy applications. The modeling of that source of error in the analysis of SLR data comprises the determination of the delay in the zenith direction and subsequent projection to a given elevation angle, using a mapping function. Standard data analyses practices use the Marini-Murray model for both zenith delay determination and mapping. This model was tailored for a particular wavelength and is not suitable for all the wavelengths used in modern SLR systems. Using ray tracing through a large database of radiosonde data, we assess the zenith delay models and mapping functions currently available and the sensitivity of models and functions to changes in the wavelength and we give some recommendations towards a unification of practices and procedures in SLR data analysis.

1. **Introduction**

Atmospheric refraction is an important accuracy-limiting factor in the application of satellite laser ranging (SLR) to high-accuracy geodetic and geophysical applications and in robust combination of solutions from other space techniques.

The propagation refraction due to the atmosphere, d_{atm}, is given by

$$d_{atm} = \int_{ray} (n-1)\, ds + \left[\int_{ray} ds - \int_{vac} ds \right], \tag{1}$$

where the first term on the right-hand side is the excess path length due to the delay experienced by the signal propagating through the atmosphere, i.e. the propagation delay, and the term in parentheses is the geometric delay, corresponding to the difference between the refracted and the rectilinear ray paths, also known as ray bending.

For modeling purposes, the atmospheric refraction can be explicitly written as the contribution of a hydrostatic and a wet component, each one consisting of the product of the delay experienced in the zenith direction and a mapping function that models the elevation angle dependence of atmospheric refraction:

$$d_{atm} = d_h^z \cdot m_h(\varepsilon) + d_w^z \cdot m_w(\varepsilon), \tag{2}$$

where d_{atm} is the atmospheric refraction at a given (unrefracted) elevation angle ε, d_h^z and d_w^z are the hydrostatic and wet zenith delays, and $m_h(\varepsilon)$ and $m_w(\varepsilon)$ are the hydrostatic and wet mapping functions, respectively.

Alternatively, a single mapping function can be considered; in this case, we have:

$$d_{atm} = d_{atm}^z \cdot m(\varepsilon)\,, \tag{3}$$

where d_{atm}^z is the zenith total propagation delay and m(ε) the (total) mapping function.

The propagation delay experienced by a laser signal in the zenith direction is defined as

$$d_{atm}^z = 10^{-6} \int_{r_s}^{r_a} N\, dz\,, \tag{4}$$

where N is the group refractivity, r_s is the geocentric radius of the laser station, r_a is the geocentric radius of the top of the neutral atmosphere, and dz has length units.

As recommended by the International Association of Geodesy [IUGG, 1999], the group refractivity for visible and near infrared waves should be computed using the procedures described in Ciddor [1996] and Ciddor and Hill [1999]. The approximate closed formula for the group refractivity in ambient moist air is:

$$N = (n-1)\times 10^6 = \left(\frac{273.15}{1013.25}\frac{P}{T} N_g\right) - 11.27\frac{e}{T}\,, \tag{5}$$

where

$$N_g = (n_g - 1)\times 10^6 = 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4}\,, \tag{6}$$

P is the total pressure (hPa), e is the partial water vapor pressure (hPa), T is the temperature in the ITS-90 temperature scale (K), λ is the carrier wavelength of the signal (μm), and N_g is the group refractivity of standard air with 0.0375% CO_2 content at T = 273.15 K, P = 1013.25 hPa, and e = 0.0 hPa.

The water vapor pressure (in hPa) is computed using the following expression [Giacomo, 1982; Davis, 1992]:

$$e = 0.01 \exp\left(1.2378847\times 10^{-5} T^2 - 1.9121316\times 10^{-2} T + 33.93711047 - 6.3431645\times 10^{3} T^{-1}\right) \tag{7}$$

The formulations expressed by Eq. (2) and (3) are very convenient and help in the identification of different components in the atmospheric refraction error budget: errors in zenith delay determination and errors in mapping functions. Furthermore, as the neutral atmosphere is a dispersive medium for the optical wavelengths, we have to consider the wavelength-dependency of the zenith delay and the mapping function.

In most of SLR data analysis, the correction of the atmospheric refraction is still performed using the Marini and Murray [1973] model. This model is based on the formulation expressed by Eq. (3), but with no clear separation of the zenith delay and the mapping function (for discussion purposes we designate this type of model as “full model”); the formulation expressed by Eq. (2)

was used by Saastamoinen [1973] and Yan and Wang [1999] in the development of zenith delay models and/or mapping functions. The performance of these models and the mapping function developed by Mendes et al. [2002] is discussed in the next sections.

2. Zenith delay model assessment

For this study, the zenith delay models of Marini-Murray, Saastamoinen and Yan-Wang were selected, and the analysis is limited to the total delay only (that is, for Saastamoinen and Yan-Wang models, we lumped together the error in the hydrostatic and the wet component). Our benchmark values were obtained using ray tracing one year (1998) of radiosonde profiles for a large number of stations (see Figure 1).

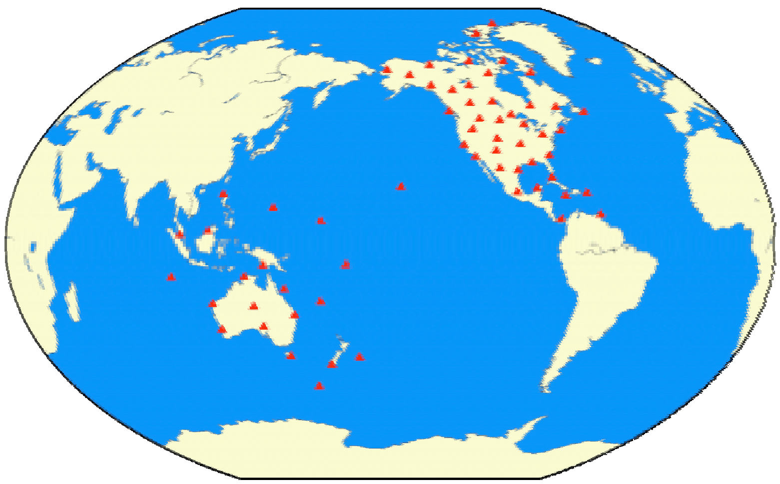


Figure 1 – Location of radiosonde stations.

Raytracing was performed using the group refractivity given by the computer procedures described in Ciddor [1996] and Ciddor and Hill [1999]. Water vapor pressure was computed using the formulation described by Davis [1992]. In order to evaluate how well the models account for changes in wavelength, we performed the ray tracing for the most commonly used wavelengths in SLR systems: 355 nm, 423 nm, 532 nm, 847 nm, and 1064 nm. The results of this assessment study are shown in Table 1.

We can conclude that all models show similar rms values of about 1 mm (the results for 532 nm wavelength agree with those published in Mendes et al. [2002], for a much larger database; note that in that paper, the units for Table 3 should be mm and not cm). The performance of the models is particularly poor for the 355 nm wavelength, with rms values of a few millimeters, particularly for the Saastamoinen model. These differences might be essentially due to limitations in the dispersion factor, but such fact has not been investigated.

Table 1 - RMS for zenith delay models, in the sense "model minus ray tracing" (mm)

λ (nm)	Marini-Murray	Saastamoinen	Yan-Wang
355	4.2	7.6	4.0
423	0.8	1.6	0.7
532	1.2	1.2	1.4
847	1.2	1.2	1.4
1064	1.1	0.9	1.3

3. Mapping function assessment

Most of mapping functions used in SLR data analysis are tailored for a given wavelength; the only exception is the mapping function by Yan and Wang [1999]. Using the same database of benchmark values, for 3 different elevation angles (15º, 10º and 6º), we assessed the performance of the mapping functions of Saastamoinen [1973], Marini and Murray [1973] and Mendes et al. [2002]; from Mendes et al. we selected FCULa (mapping function only) and FCULz (FCULa combined with the Saastamoinen zenith delay model). The results of this assessment for 10º elevation angle are shown in Table 2 (note that Marini-Murray and FCULz include the zenith delay prediction, whereas Yan-Wang, Saastamoinen and FCULa show errors in mapping functions only).

Table 2 - RMS for Mapping Functions (ε = 10º), in the sense "model minus ray tracing" (cm)

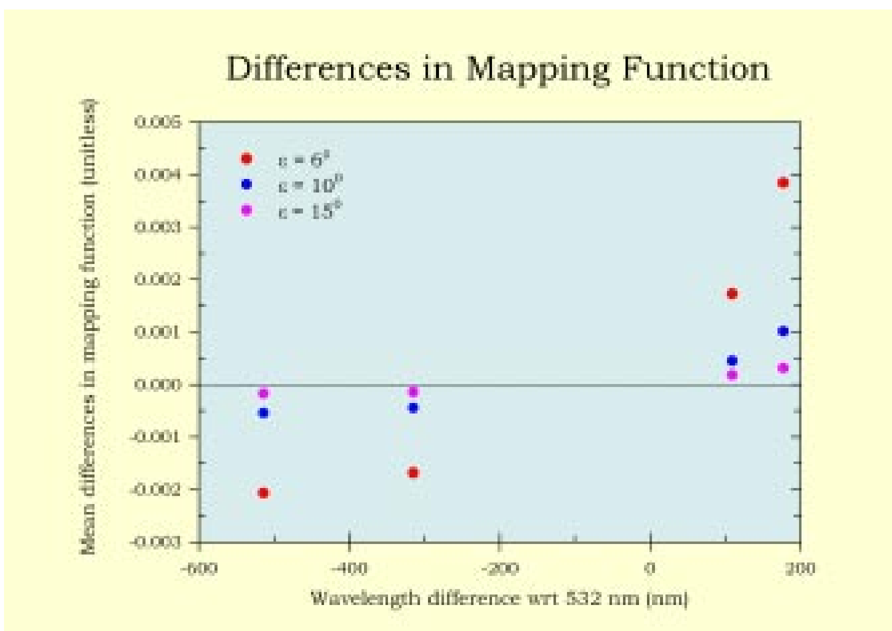
λ (nm)	**Full models**		**Mapping functions ONLY**		
	Marini-Murray	FCULz	FCULa	Yan-Wang	Saastamoinen
355	1.77	3.83	0.55	1.72	3.04
423	0.79	0.75	0.46	1.65	2.48
532	1.14	0.82	0.41	1.56	2.16
847	1.05	0.75	0.39	1.56	2.05
1064	0.98	0.62	0.39	1.77	2.06

The results for this elevation angle show that the effect of the neglecting the wavelength dependence of the mapping function is not critical; as in the case of the zenith delay models, the degradation of the models' performance is larger for lower wavelengths and also affects the Yan and Wang model, despite the explicit inclusion of the wavelength dependence in the formulation. Due to the larger error associated to the Saastamoinen zenith delay model, the error for FCULz is larger than the error for Marini-Murray for λ = 355 nm. The changes in mapping factor with respect to the 532 nm wavelength are at most ~0.004, for an elevation angle of 6º (see Figure 2), representing only ~1 cm error, for a nominal value of 2.4 m zenith delay, which is below the precision level of any of the mapping functions. However, due to the systematic effect that it induces, it is recommended that this effect be included in the development of future mapping functions.

4. Conclusions

We have analyzed the performance of zenith delay models and mapping functions for the most commonly used wavelengths in SLR systems. We have concluded that all zenith delay models have errors at the millimeter level, which increase significantly at 355 nm. The wavelength dependence of the mapping function is not significant for elevation angles above 10º and is

Figure 2 – Mean differences in mapping function with respect to 532 nm wavelength.



within the precision level of current mapping functions. However it is a systematic effect and it is recommended that future developments in mapping functions should take this effect into account.

Acknowledgments

Radiosonde data was kindly provided by the British Atmospheric Data Center (BADC). The ray-trace software was developed by James Davis, Thomas Herring, and Arthur Niell.

References

Ciddor, P.E. (1996). “Refractive index of air: New equations for the visible and near infrared.” *Applied Optics*, Vol. 35, No. 9, pp. 1566-1573.

Ciddor, P.E. and R.J. Hill (1999). “Refractive index of air. 2. Group index.” *Applied Optics*, Vol. 38, No. 9, pp. 1663-1667.

Davis, R.S. (1992). “Equation for the determination of the density of moist air.” *Metrologia*, Vol. 29, pp. 67-70.

Giacomo, P. (1982). “Equation for the determination of the density of moist air (1981).” *Metrologia*, Vol. 18, pp. 33-40.

International Union of Geodesy and Geophysics (IUGG) (1999). Resolution 3 of the International Association of Geodesy, *Comptes Rendus of the XXII General Assembly*, 19-30 July 1999, Birmingham, pp. 110-111.

Marini, J.W. and C.W. Murray (1973). “Correction of laser range tracking data for atmospheric refraction at elevations above 10 degrees.” NASA-TM-X-70555, Goddard Space Flight Center, Greenbelt, Md.

Mendes, V.B., G. Prates, E.C. Pavlis, D. E. Pavlis, and R.B. Langley (2002). “Improved mapping functions for atmospheric refraction correction in SLR.” *Geophysical Research Letters*, Vol. 29, No. 10, 1414, doi:10.1029/2001GL014394.

Saastamoinen, J. (1973). “Contributions to the theory of atmospheric refraction”. In three parts. *Bulletin Géodésique*, No. 105, pp. 279-298; No. 106, pp. 383-397; No. 107, pp. 13-34.

Yan, H. and G. Wang (1999). “New consideration of atmospheric refraction in laser ranging data.” *Monthly Notices of the Royal Astronomical Society*, Vol. 307, pp. 605-610.