Atmospheric Contribution to the Laser Ranging Jitter

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General

The ultimate goal of the satellite laser ranging is the millimeter precision and accuracy. To achieve this goal, all the individual contributors to the ranging error budget must be well below millimeter level. The best existing ground based ranging systems are capable to achieve millimeter ranging precision when ranging to short distance terrestrial targets. However, ranging to Earth orbiting satellites the best precision obtained is typically 3 time worse, about 3 millimeters RMS. As this value is obtained even for satellite targets not spreading the echo signal, there is a speculation, that the remaining contribution to the random error budget is contributed from the atmosphere. The ground targets ranging experiments demonstrated an increase of the ranging jitter by a factor of 0.9 psec for 100 meters atmospheric path increase [12] thus supporting this idea, as well.

We have used the available the atmospheric modelling code to simulate the propagation of the optical signal in the satellite laser ranging experiment.

Modelling code

The commercial version of the GLAD code has been used [2,3,4]. It is an extensive program for modelling of diffractive propagation of light through various media and optical devices. The light is considered to be monochromatic and coherent (or partially coherent). The electromagnetic field in GLAD is described by its two dimensional transversal distribution. Two complex computer arrays (one for each polarisation state) represent the complex amplitude (intensity and phase) at each point in *x* and *y* axis.

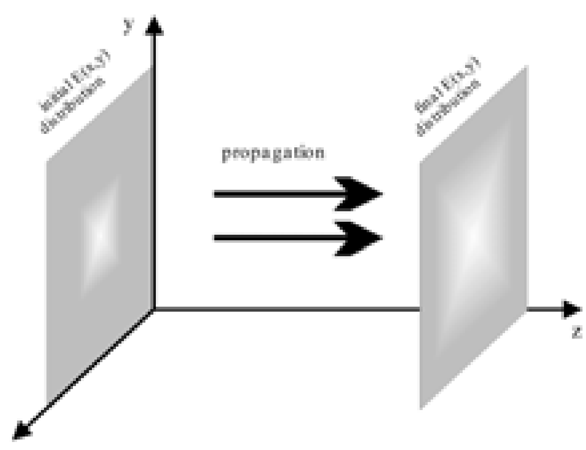

Fig. 1 – Scheme of light propagation model used in GLAD

The propagation is done by the *angular spectrum* method. That means, the field distribution is decomposed into a summation of plane waves, these plane waves are propagated individually and then resumed into resulting distribution.

Atmospheric Aberration and Its Parameters

Refractive turbulence in the atmosphere can be characterized by three parameters. The *outer scale parameter* L_o is the size of the largest turbulent eddies and its value is usually on the order of meters [5, 6]. The *inner scale parameter* L_i is the length of the smallest eddies and is usually about 5–10 mm [5, 3]. In our computations, we used the following values: $L_i = 0.1$ cm, $L_o = 1$ m.

The third parameter C_n is called *refractive index structure constant* and characterises the strength of turbulence at certain place in the atmosphere. It is defined by equation:

$$\langle [n(r_1) - n(r_2)]^2 \rangle = C_n^2 (r_1 - r_2)^{2/3} \tag{1}$$

where $n(r)$ is the refractive index at point r and the angle brackets symbolise an ensemble average [5]. This equation is valid for $L_i < (r_1 - r_2) < L_o$.

In case of computations of horizontal light propagation near the ground, we used a typical value of $C_n = 3.10^{-13}$ $\mathrm{m}^{-2/3}$. In case ground-to-space propagation, Hufnagel-Valley model of height dependence of C_n was used:

$$C_n^2 = 3.59 \cdot 10^{-3} (h \cdot 10^{-5})^{10} \exp\left(-\frac{h}{1000}\right) + 2.7 \cdot 10^{-16} \exp\left(-\frac{h}{1500}\right) + + A \exp\left(-\frac{h}{100}\right) \tag{2}$$

where h is height above the sea level in meters and the constant A is commonly set to 1.7e-14.

In GLAD, the integral strength of atmospheric turbulence along a path is characterized by Fried's parameter r_0, which is sometimes called also "Fried's coherence length" and can be defined as the distance of two points on randomly distorted wavefront, where the phase difference RMS increases to 2.62 rad. It was computed from C_n by:

$$r_0 = \left[\frac{16.5}{\lambda^2} \int_{x_0}^{x_1} C_n^2(x) dx \right]^{-3/5} \tag{3}$$

where λ is wavelength of the light and x is distance along the propagation path.

Chromatic Dispersion Influence

One of the questions was, if chromatic dispersion can affect the distribution of random path differences of the pulses. The idea was, that the different colour components of the pulse will propagate through slightly different states of atmosphere and could be differently delayed.

If we compute a characteristic path difference of particular colour components of a short pulse after several kilometers of atmospheric propagation, we obtain a value of tens of micrometers, that means time difference of several hundreds of femtoseconds. However, characteristic time of evolution of atmospheric turbulent structures is much larger (milliseconds or more – from wind speed and size of the smallest turbulent eddies). Hence, all the colour components of picosecond laser pulse "see" the same state of atmosphere when propagating through the air and *the chromatic dispersion need not to be considered* when modelling random fluctuations of pulse delay.

Atmospheric Aberration Modelling in GLAD

The computations that we made consisted of following steps:

- Specification of initial conditions – array size, wavelength and initial intensity and phase distribution (plane wave or gaussian beam).
- Computation – alternating steps of aberration addition and diffraction propagation (recommended by [3]). Each aberration is characterised by the value of r_0.
- Resuming the results – statistics of the wavefront shift at different points, computing of the beam centroid, computing of the beam radius, saving the data etc.

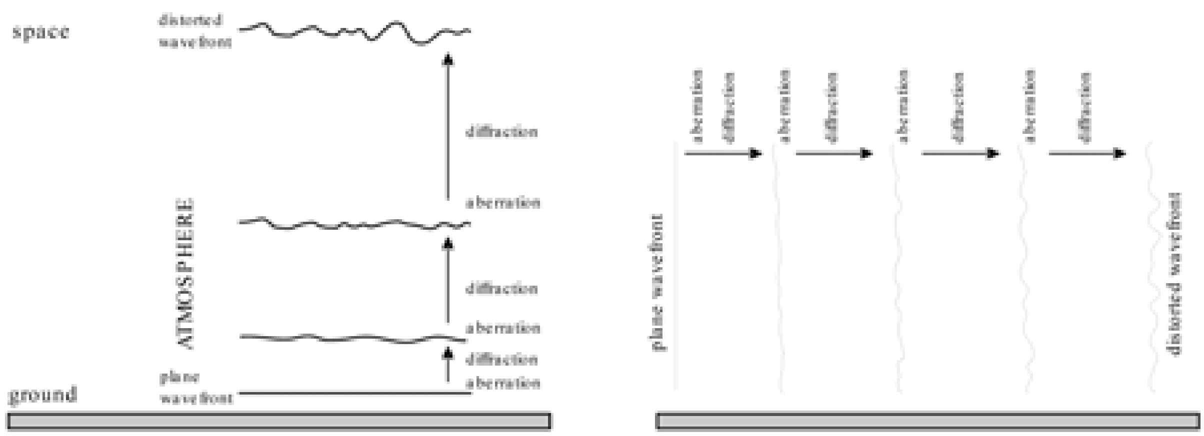

Fig. 2 – Schemes of our ground-to-space and horizontal propagation models.

In GLAD, the random wavefront aberration modelling is based on Lutomirski-Yura model of wavefront power spectrum [3]:

$$W^2(\rho) = \frac{0.023 \cdot e^{-\rho^2 L_s^2}}{r_0^{5/3}\left(\rho^2 + \frac{1}{L_s^2}\right)^{11/3}} \tag{4}$$

where ρ is the spatial frequency variable. Using this equation and user-specified value of r_0, random aberration is computed and added to the wavefront.

Computed Results

At first, we decided to check the functionality and applicability of our model. We tried to predict the angle of astronomical seeing by our model and compare it with value coming from following equation (taken from [9]):

$$\theta_{FWHM} = \left(\frac{1.009 \cdot D}{r_0}\right)^{5/6} \frac{\lambda}{D} \quad (5)$$

where θ_{FWHM} is the seeing angle – size of the apparent disk of a star, photographed using long exposure by a telescope of diameter D at wavelength λ, when the atmospheric turbulence conditions are characterised by the Fried's parameter r_0.

Let r_0 = 5.2 cm and D = 1 m. Then equation (5) gives θ_{FWHM} = 1.22 arcsec. Our model (atmospheric propagation of a plane wave from space to ground, application of 1-m aperture and focusing, averaging of 100 resulting profiles for different random seeds, computing FWHM) gives θ_{FWHM} = 1.31 arcsec, which is in good agreement with the theoretical value. Also the results we computed for smaller telescope apertures (down to 8 cm) agree well with eq. (5).

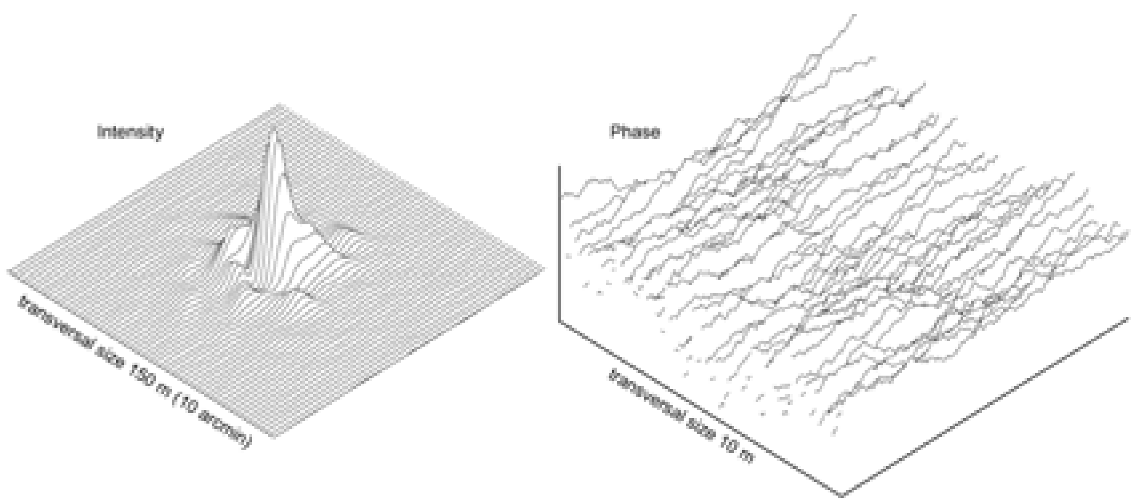

Fig. 3 – Illustrations of resulting intensity and phase profile after propagation through atmosphere, computed using GLAD (the starting distribution was gaussian beam, total path length 3000 km)

Ground-to-space and horizontal propagation models

In our models of laser pulse propagation, we assumed wavelength of 500 nm. Initial field distribution was specified as a section of a plane wave of 10 m in diameter and array size was 256x256 (that means one element had a size of 3.9 cm).

In case of ground-to-space propagation, the field distribution was propagated from the sea level to height of 300 km in zenith angle of 45 deg. The propagation was done in three spatial steps of different length and the corresponding values of r_0 were computed from the equations

(2) and (3) and the integral value of r_0 was about 4 cm (equivalent to seeing of about 1.5 arcsec for 1-m aperture).

In case of horizontal propagation, the initial distribution was propagated to distance of 6 km through atmosphere with total r_0 of 0.1 cm (comes from typical near-ground value of C_n as described above). The propagation was done in 50 steps, each 120 m long and with r_0 of 1.04 cm.

In both cases of propagation, 100 passes for different random seeds were computed (that means 100 different random aberrations of the same strength, given by r_0), and from the 100 resulting phase profiles of the wavefronts we made statistics to find out the distribution of random phase shifts.

In both cases, the distribution was gaussian, as shown at Fig. 4. For better visualization, we recomputed the phase shifts $\Delta\varphi$ to spatial shifts Δs as

$$\Delta s = \frac{\Delta\varphi}{2\pi}\lambda$$

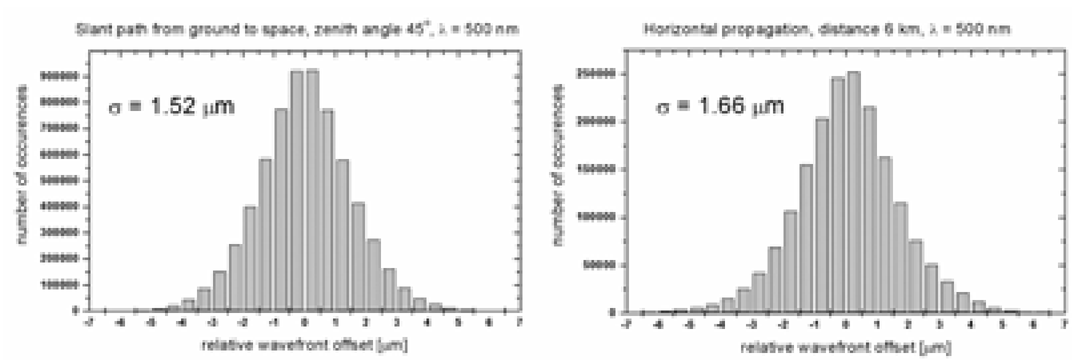

Fig. 4 – Results of our ground-to-space and horizontal propagation models.

It is obvious, that the random differences of a picosecond pulse's optical path, caused by atmospheric turbulence, are very small – 10 micrometers peak to peak maximum. It means that the turbulence cannot be responsible for the satellite laser ranging jitter increase.

This conclusion is supported by several independent checks:

- near-ground interferometric measurements [6, 11] resulted in similar phase and path random distribution as our model
- in adaptive optics, used i.e. in astronomy, the big adaptive mirrors are naturally not able to correct optical path differences of the distorted wavefront, that are larger than several micrometers - this fact just corresponds to our results

Acknowledgement

This research has been supported by the grant MSM 210000015 provided by Ministry of Education of the Czech Republic.

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