Modification of Laser Ranging Equation

Xiong Yaoheng[*] Feng Hesheng[*]
Yunnan Observatory, National Astronomical Observatories, CAS
Kunming 650011, P.R.China

ABSTRACT

The goal of this paper is to discuss the atmospheric turbulence effects on the laser ranging, especially for the lunar laser ranging. Concerning the returned laser photons from the space retroreflector for a laser pulse firing from the ground telescope, the laser beam wander that is caused by the turbulence plays an important role in it. Considering the short-term beam wander and the Gaussian distribution of the laser beam along radial, a new form of the laser ranging equation is given. If the atmospheric tilt, that corresponds to the beam wander, is removed, the returned photon numbers will be increased a factor of 6 to 40, depend on the turbulence.

Keywords: Atmospheric turbulence, Laser beam wander, Returned photon numbers

1. INTRODUCTION

The classical laser ranging equation, that was used to count the returned laser photons reflected from the space retroreflector for one laser pulse transmission from the ground telescope, did not consider the atmospheric turbulence effects on it. For current SLR technologies, enough returned photons can be obtained, even the ranging accuracy will be affected with the amount of less than 1mm for most condition by the atmospheric turbulence, it is no need to consider the atmospheric turbulence effects on SLR. For LLR, the received laser photons for one laser pulse firing by a 1m telescope on the ground are less than 1, so it is needed to pay an attention to where the other photons go.

When a laser beam propagates through the atmosphere, its quality will be affected by the turbulence. As consequences, the laser beam will have following distorted terms: random time delay, pulse spread, beam wander, spread and scintillation. Most turbulent effects can be neglected for the laser beam on the laser ranging. But the short- term wander that is caused by the atmospheric tip-tilt will result in the laser beam deviation from the retroreflectors on the moon surface. So in this paper, the short- term beam wander and the Gaussian distribution of the laser beam along radial are considered in the new form of the laser ranging equation.

In section 2, there are the returned laser photons according to the classical laser ranging equation, and from this equation some key factors that are neglected now can be found. In section 3, the atmospheric turbulence effects on laser beam propagation and laser ranging are discussed. The last section is a new idea to compensate the atmospheric tip-tilt in real-time for the laser beam on the LLR and a technical plan that is possible for Yunnan Observatory 1.2m laser ranging system.

[*]contact. yozsx@public.km.yn.cn; phone 86 871 3911347; fax 86 871 3911845; Yunnan Observatory, National Observatories, Chinese Academy of Sciences, Kunming 650011, Yunnan, P.R.China

2. RETURNED PHOTON NUMBERS ON LR

In laser ranging technology, a pulse laser beam will be transmitted from a ground telescope to a space target, either satellites or the moon with retroreflectors, and after reflecting the returned laser photons will be back to the ground telescope. Counting the start and the stop time of one laser pulse accurately, the ranging between the ground telescope and the space target will be measured with an accuracy of 1~2 cm. If the laser beam is considered as uniform and propagation along a geometrical line, the returned photoelectron numbers N for one laser pulse can be obtained from the classical laser ranging equation:

$$N = \frac{16EN_0 A_m A_r T_a^2 T_t T_r \eta\alpha}{\pi^2 R^4 \theta_e^2 \theta_m^2} \qquad (1)$$

For one laser pulse firing of Kunming 1.2m laser ranging system and for Apollo 15 retroreflector:

N=0.17

Because of the Poisson distribution of the photoelectrons, the probability of one photoelectron that we can get from one laser pulse on the LLR is:

$P(1)=1-e^{N}=0.16 \qquad (2)$

The A_m of Apollo 11 and Apollo 14 are three times less than that of Apollo 15, and the A_m of Lunakhod 2 is about 4.6 times less than that of Apollo 15, so we can image the probability of one photoelectron to be detected.

Obviously, in the process of deduction of the classical laser ranging equation, two factors have not been considered: atmospheric turbulence effects on laser beam propagation, and the distribution of the laser beam along its radial.

3. ATMOSPHERIC TURBULENCE EFFECTS ON LASER PROPAGATION AND LASER RANGING

3.1 Atmospheric turbulence effects on laser beam propagation

When a laser beam propagates through the atmosphere, because of a random movement of the atmospheric turbulence, the index of refraction of the Earth's atmosphere has a fluctuation. That results in a series of effects on the laser beam: random time delay, pulse spread, beam wander, beam spread and scintillation.

Among these random atmospheric effects, the time delay and pulse spreading can be negligible [1] [2] (<1 ps).

The scintillation, that is the intensity fluctuation, has no significant effect on the laser ranging (variance of intensity fluctuation ≤ 0.02) [3].

For the laser ranging, we consider only the beam wander and beam spread, especially for the short-term items.

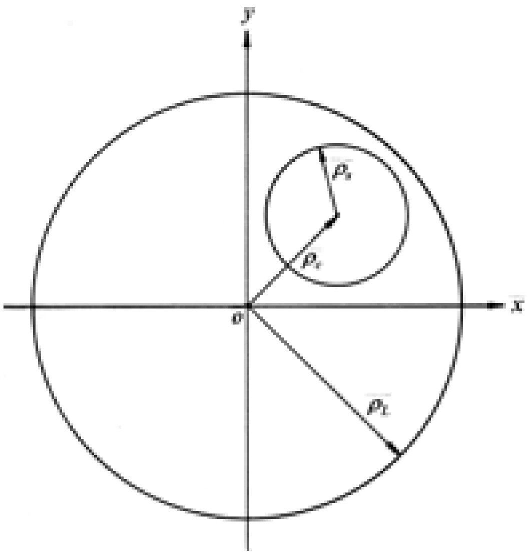


Fig.1 Laser beam at far field

Fig.1 is the diagram to show a laser beam at far distance when it has been affected by the turbulence. If there is no turbulence, the laser spot at far distance is determined by its transmitted divergence and is at its pointing center *o*. But the real condition is that the laser beam has a short-term beam wander and a short-term spread with the radius ρ_C , ρ_S , respectively.

For a long time average, a long-term beam spread with the radius ρ_L can be seen.

From Maxwell wave propagation equation and using Markov approximation (index of refraction fluctuation is a delta-function correlated in direction of propagation) to calculate the ensemble averaged quantity of the field, the second moment and the fourth moment of the field [4]. Some statistic quantities to describe the atmospheric effects for the laser beam can be obtained.

Short-term beam wander:

$$\langle \rho_C{}^2 \rangle = \frac{10.22 z^2}{k^2 r_o^{\frac{5}{3}} D^{\frac{1}{3}}} \qquad (3)$$

Short-term beam spread:

$$\langle \rho_S{}^2 \rangle = \frac{4z^2}{k^2 D^2} + \frac{D^2}{4}\left(1 - \frac{z}{F}\right)^2 + \frac{17.6 z^2}{k^2 r_o{}^2}\left[1 - 0.48\left(\frac{r_o}{D}\right)^{\frac{1}{3}}\right]^{\frac{6}{5}} \qquad (4)$$

Long-term beam spreading:

$$\langle \rho_L{}^2 \rangle = \frac{4z^2}{k^2 D^2} + \frac{D^2}{4}\left(1 - \frac{z}{F}\right)^2 + \frac{17.6 z^2}{k^2 r_o{}^2} \qquad (5)$$

Here,

k wave number,

D laser transmitter diameter

z laser propagation axis and coordinate

F radius of curvature of laser beam

r_o Fried's coherence length

Changing ρ_C , ρ_S , and ρ_L to their correspond angel θ_C , θ_S , and θ_L , for a distance, following data can be obtained with different Fried's coherence lengths. Following are their values.

	r_o=5cm	r_o=10cm	r_o=15cm
θ_L	2.″93	1.″48	0.″98
θ_S	2.″63	1.″27	0.″83
θ_C	1.″32	0.″74	0.″53

Table 1. Atmospheric effects on the laser beam at different r_o

Above θ_C , θ_S , and θ_L add to the laser beam and have a significant influence on the beam propagating direction and on the beam shape. For low-order real-time correction, only short-term beam wander is considered.

3.2 Atmospheric turbulence effects on laser ranging

3.2.1 Laser ranging accuracy

There are some classical corrective formulas of laser range tracking data for atmospheric refraction at elevation above 10 degrees [5], without considering the turbulence random movement for the index of refraction.
Consideration a random path deviation caused by the refractive index fluctuation for a round trip laser ranging, then calculation the covariance of the average path deviation, the accuracy of the laser ranging ΔL is obtained [6]:

$$\langle \Delta L^2 \rangle = \frac{3.127 C_n^{\ 2}(0) L_o^{\frac{5}{3}} h_T}{SinE} \qquad (6)$$

Here,
$C_n^{\ 2}$ turbulence structure parameter L_o turbulence outer scale
h_T atmospheric scale height E target elevation angle
For different seeing conditions and elevations, following data are obtained on Tab.2:

$<\Delta L^2>^{1/2}$ (mm)

	E=10^0	E=30^0	E=60^0
$C_n^{\ 2}$~10^{-13} m$^{-2/3}$	10.33	6.09	4.63
$C_n^{\ 2}$~10^{-15} m$^{-2/3}$	0.83	0.45	0.37
$C_n^{\ 2}$~10^{-17} m$^{-2/3}$	0.17	0.10	0.08

Table 2. Atmospheric effects on laser ranging accuracy at different elevations

If the seeing condition is good at an observation place, the accuracy of the laser ranging affected by the atmospheric turbulence can be negligible.

3.2.2 Returned laser photons

On the satellite or the lunar laser ranging, retroreflectors on these space targets are needed. It may be called passive laser ranging, that is different from some newly proposed space laser ranging mission, such as compact laser transponder for interplanetary ranging between Mars and Earth [7], or ASTROD [8]. It is active laser ranging, in which a laser will be emitted from the space target to the ground station when it receives a laser signal from the ground. For the passive laser ranging, the returned photons are important. It determines whether this laser ranging will succeed.

If the short-term laser beam wander caused by the atmospheric turbulence is considered, and the Gaussian distribution of the laser beam is also considered, the modification of the laser ranging equation (returned laser photoelectrons on the ground receiver: N_r) is obtained:

$$N_r = \frac{4EN_o A_m A_r T_a{}^2 T_t T_r \eta\alpha}{\pi^2 \left(\theta_e{}^2 + \theta_s{}^2\right)\theta_m{}^2 R^4} \exp\left(-\frac{\rho_C{}^2}{\rho_e{}^2 + \rho_s{}^2} \right) \tag{7}$$

The meaning of some special parameters are:

θ_s laser beam divergence caused by the short-term wander

ρ_e laser beam diameter on target, determined by θ_e

For one laser pulse on lunar laser ranging and for 1.2m laser ranging system:

N_r=0.17×(1/40 ~ 1/6)

More less than one photoelectrons! We can call it is a sub-single photon detection!

If tilt is removed, the correction factor for the laser ranging is:

$$\frac{N_r}{N} = \frac{\rho_e{}^2}{4(\rho_e{}^2 + \rho_s{}^2)} \exp\left(-\frac{\rho_C{}^2}{\rho_e{}^2 + \rho_s{}^2} \right) \tag{8}$$

About 1/6 ~ 1/40, depend on the turbulence. That is if the atmospheric tilt is removed, the returned photoelectrons will be increased a factor of 6 to 40.

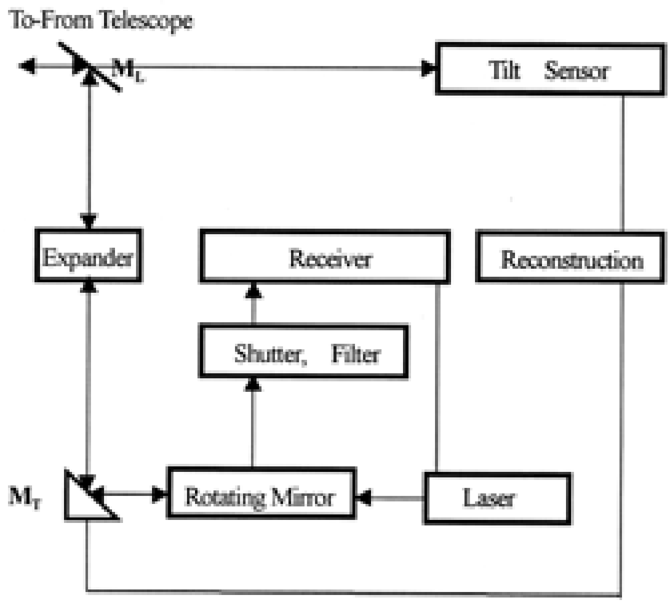
4. FURTHER THOUGHTS

When returned laser photons are much less than 1 for one laser pulse firing, it is needed to consider a technical plan to increase the returned photons. For current laser ranging, many aspects for increasing the returned photons have been considered. Using adaptive optics technique in the laser ranging is one way and proposed [9], especially for the LLR. Its purpose is to compensate atmospheric turbulence effects in real time on the laser ranging.

Because atmospheric tilt results in the laser beam wander, and near 87% wavefront distortion is caused by it[10], using a low order correction for the LLR that is easy to realize is suggested.

For the LLR, using a wavefront sensor and the absolute difference algorithm, the wavefront tilt can be detected from the moon surface [11]. That is to calculate the absolute difference values between a live image and a reference image those are taken from a same small area near the moon retroreflector in time sequence. Next is to separate the atmospheric tilt from these values, then to drive a tip-tilt mirror for the real-time tilt compensation (~ ms) for the laser beam that will be emitted on the LLR.

For Kunming 1.2m laser ranging system, a tilt detection part and a tip-tilt mirror M_T will be added to the system. That will perform the real-time tilt compensation for the uplink and the



downlink laser beam on the LLR. Fig.2 is a diagram for this technical plan.

Before starting a laser beam on the LLR,, the tilt sensor is used to detect a series of images of the interested area near the moon retroreflector within the isoplanatic angle. M_L is a dichroic mirror that reflects laser wavelength and passes other lights. After reconstruction wavefront, a tip-tilt signal is sent to the tilt mirror M_T to perform a real-time tilt compensation for an uplink laser beam that is to emit simultaneously. That will let the Gaussian laser beam hit the moon retroreflector accurately and centrally and more laser photons return to the ground telescope. According to the time gate, the same step is also taken for the downlink laser beam. After pass the M_T, some returned laser photons that is offset the telescope optical axis will enter the receiver area totally.

Fig.2 Coudé optical layout of 1.2m LR system

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