Prospects for an Improved Lense-Thirring Test with SLR and the GRACE Gravity Mission

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Abstract

The theory of General Relativity predicts several non-Newtonian effects that have been observed by experiment, but one that has not yet been directly confirmed with confidence is the existence of the Lense-Thirring precession of an orbit due to the gravitomagnetic field. Previous analyses using satellite laser ranging (SLR) data to LAGEOS-1 and LAGEOS-2 are limited by optimistic and unprovable assumptions regarding the magnitude and correlation of the errors in the low degree geopotential harmonics. Now that the joint NASA-DLR GRACE (Gravity Recovery and Climate Experiment) mission has already determined a dramatically improved geopotential model, we can examine the expected improvements in the Lense-Thirring experiment.

Introduction

In General Relativity, the presence of mass causes 4-dimensional space-time to curve (Figure 1). A satellite that would travel in a straight line on the flat Minkowskian space (i.e., unaccelerated) still travels a straight line on the curved manifold but only in the local sense. To an external observer, the path of the satellite looks curved and is interpreted as orbital motion.

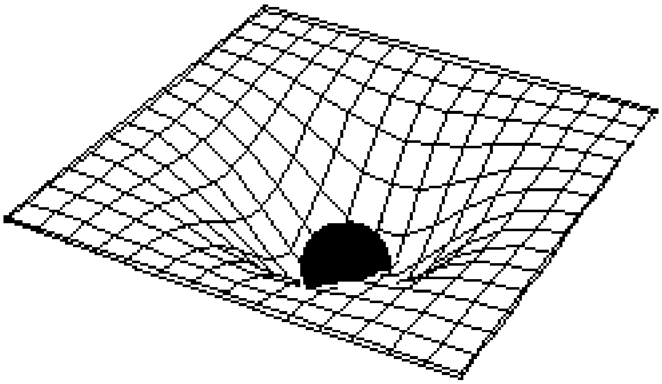

Figure 1. Space in the presence of an object with mass.

However, when the influencing body is rotating, the local inertial frame of the satellite can be thought of as being "dragged" in the direction of rotation (Figure 2). For this reason, the effect of the central body's rotation on a satellite's orbit is often called "frame-dragging" [*Lense and Thirring*, 1918]. It has also been called gravitomagnetism through a different line of reasoning [*Ciufolini*, 1986]. Just as a spinning charge produces a magnetic field, a spinning mass (angular momentum) causes the creation of mass currents and hence the production of a gravitomagnetic field (Figure 3).

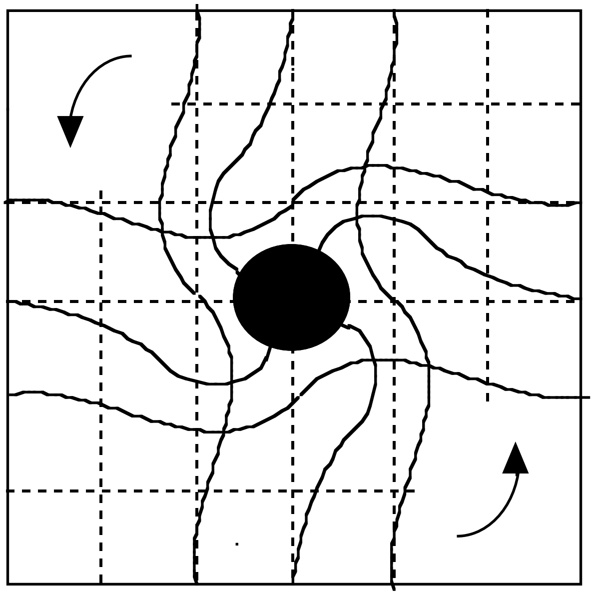

Figure 2. Influence of rotation on the background space-time.

$$\bar{F}_B = q_0 \bar{v} \times \left[\nabla \times \left(\frac{\bar{M} \times \bar{r}}{r^3} \right) \right] \qquad \bar{F}_{LT} = m_0 \bar{v} \times \left[\nabla \times \left(\left(\square (1+\gamma) \right) \frac{G}{c^2} \frac{\bar{J}_{cb} \times \bar{r}}{r^3} \right) \right]$$

Spinning Electron Spinning Mass

Figure 3. Similarity between electromagnetic and gravitomagnetic fields (M is the magnetic moment and J_{cb} is the angular momentum of the central body, q_0 and m_0 are the charge and rest mass, r and v are the position and velocity, G and c are the gravitational constant and speed of light, γ is a relativity parameter).

Observing the effect of the gravitomagnetic field is important for several reasons. First, this field can be considered as a new field of nature, which is analogous to the magnetic field in electrodynamics. Second, the measurement of the gravitomagnetic field will provide experimental support for the general relativistic formulation of the Mach principle that the local inertial frames are determined or at least influenced by the mass-energy distribution and currents in the universe [*Wheeler*, 1988]. Finally, a demonstration of this effect would be of significant importance for high-energy astrophysics. Some theories of energy storage, power generation, jet formation and jet alignment of quasars and active galactic nuclei are based on the existence of the gravitomagnetic field of a supermassive black hole [*Thorne et al.*, 1986]

Testing for Gravitomagnetism

Gravitomagnetism has two observable effects: the Schiff precession and the Lense-Thirring precession. Both can be described as the "dragging of inertial frames" but have different consequences. The Schiff effect arises from the spin-spin interaction between the satellite's intrinsic angular momentum (i.e., its spin) and the angular momentum of the central body (central body spin). Both spinning bodies are producing a gravitomagnetic field but the larger central body's field is forcing a change in the local inertial frame which can be observed through monitoring the change in the smaller body's field. Equivalently, this means that the Schiff effect causes a precession in the spin axis of an orbiting satellite (or gyroscope). The NASA Relativity Experiment, Gravity Probe-B [*Everitt et al.*, 1980], is designed to measure this effect through the monitoring of the precession of onboard ultra-precise gyros to better than 1% (Figure 4).

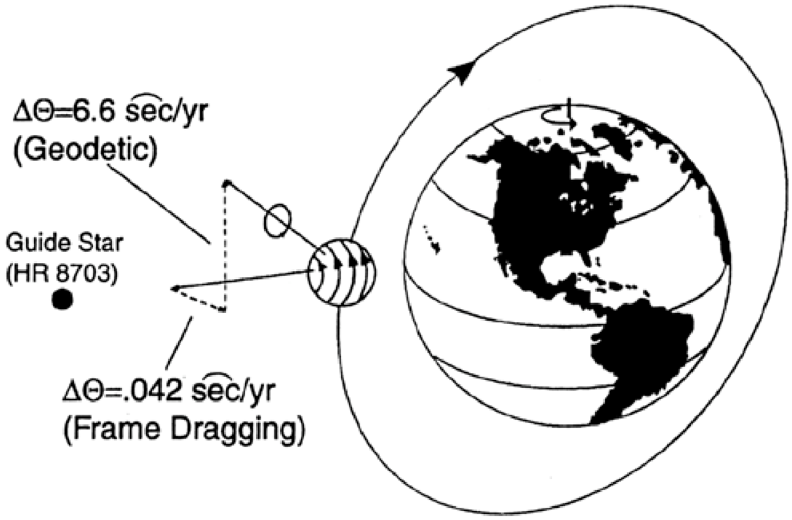

Figure 4. The GP-B relativity experiment by Stanford University, planned for launch in 2003, is expected to verify both geodesic precession and frame dragging to much higher precision than currently possible.

The Lense-Thirring effect on the other hand, is generated solely by the angular momentum of the central body. As the Earth spins, its "drag" on space-time produces a systematic change in the line of nodes and in the perigee, a perturbation that is equivalent in form to a zonal geopotential effect. Direct measurement of the Lense-Thirring precession is a difficult proposition. The effect is extremely small, ~31 mas/yr for the LAser GEOdynamics Satellite (LAGEOS) node [*Ciufolini*, 1986]. The dominant source of difficulty in detecting this small signal lies with the errors in the even zonal portions of the geopotential. The even zonals contribute a secular motion to the satellite's orbit which, if perfectly known, could be removed from the orbit analysis. However, the even zonals are not known exactly; the current error in the knowledge of J_2 is on the same order of magnitude as the Lense-Thirring effect. In addition to the static error portion of the gravity field, errors in the time-varying portions of the even zonals (secular, tidal and seasonal variations) also swamp the precession due to their magnitude relative to the Lense-Thirring effect.

In order to overcome these deficiencies, *Ciufolini* [1986] proposed using two LAGEOS type satellites in supplementary orbits to measure the Lense-Thirring precession where a proposed LAGEOS-3 satellite would be placed into an orbit identical with that of LAGEOS-1 but with a supplementary inclination (Figure 5). Placing the two laser-ranged satellites in this configuration provides an exact cancellation in the precession due to the zonal coefficients. The largest remaining error source, tidal effects that do not cancel, would then average out over time. *Ries* [1989] examined the LAGEOS-1/3 configuration and obtained a level of error in the Lense-Thirring recovery of ~8%. *Peterson* [1997a] revisited the analysis using more up-to-date models and estimated that an uncertainty of ~4% could be achieved. Unfortunately, it does not appear likely that a third LAGEOS satellite will be launched in the near future to support this experiment. Consequently, any analysis must be done using SLR tracking of existing satellites.

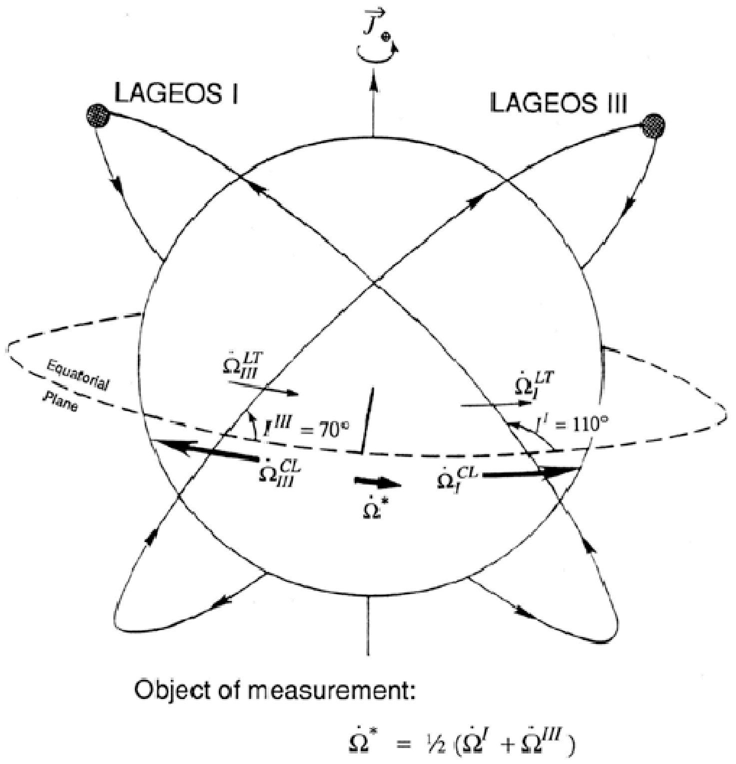

Figure 5. LAGEOS-1/LAGEOS-3 supplementary orbit configuration.

Recent Analysis

Recently, *Ciufolini et al.* [1998] asserted a confirmation of the Lense-Thirring effect at the 20% level using the EGM96 gravity model [*Lemoine et al.*, 1998] and SLR tracking to LAGEOS-1 and LAGEOS-2. Their method used three pieces of information (LAGEOS-1 node rate, LAGEOS-2 node rate and LAGEOS-2 perigee rate) to determine the LT effect along with J_2 and J_4. As a result, it was only necessary to consider the errors in the EGM96 gravity model for J_6, J_8…. This idea was innovative, but there are significant uncertainties in the assumptions used for assessing the experiment error, especially regarding the contribution of the zonals [*Ries et al.,* 1998]. One problem is the use of the LAGEOS-2 perigee to eliminate the errors in J_4. This introduces the effect of a number of non-gravitational forces for which the models are uncertain, making the subsequent error analysis difficult to prove.

A more serious problem is the use of a very favorable negative correlation between zonals in EGM96 (the result of poor separation of the zonals in the gravity solution) to reduce the error introduced by the gravity model from approximately 50% to 13%. The EGM96 covariance, like any gravity solution covariance, is only an approximate estimate of the errors in the gravity solution; it cannot be considered to be an exact representation of the magnitude or correlation of the error in the individual coefficients. Further, there is no reason to expect that the errors in the EGM96 gravity model (which is a multi-decade mean gravity solution) are representative of the actual errors in the gravity model during the period of the Lense-Thirring analysis, in light of known secular, seasonal and decadal variations in the Earth's gravity field. A more realistic error assessment would not rely on the cancellation of the errors due to a fortunate correlation, and it probably would treat the magnitude of the errors in the higher degree zonals given by the EGM96 covariance with some caution. This would lead to an estimated error in the current determination of the Lense-Thirring precession of at least 50 to 100%, if not larger.

While there may be various views about this, any experiment based on current gravity models will be plagued with uncertainties regarding the error analysis. A much more solid foundation is necessary for a confident verification of General Relativity. What is needed is a gravity model so accurate that contributions from errors in the zonals are not significant. This removes the need to use the LAGEOS-2 perigee rate, resulting in a much cleaner signal from just the node rates. In addition, any questionable assumptions about correlations between the geopotential coefficients would not be required to reduce the contribution of the errors in the gravity model.

The GRACE Gravity Mission

The Gravity Recovery and Climate Experiment (GRACE) mission was implemented under the NASA Earth System Science Pathfinder (ESSP) Project, an innovative approach for addressing global change research. A joint project between the U.S. and Germany, the delivery of the two spacecraft was contracted to Astrium GmbH under the scientific lead of the Jet Propulsion Laboratory (JPL) [*Davis et al.*, 1999; *Dunn et al.*, 2003]. The twin GRACE satellites were put into coplanar, circular orbits at an inclination of 89° by a Russian ROCKOT launcher from the Plesetsk cosmodrome on March 17, 2002. At the beginning of the mission, the orbital height was 500 kilometers above the Earth, but they will descend during the five-year mission time due to atmospheric drag.

The twin satellites in flight formation can be considered to be the effective mission instrument (Figure 6). The principle data is the High Accuracy Intersatellite Ranging System, consisting of K- and Ka-band dual-one-way range measurements of the distance between the two GRACE satellites flying in nearly identical orbits but separated by approximately 200 km. The two spacecraft experience the Earth's gravity field (and other nongravitational forces) at different positions and are thereby differentially accelerated. This difference is manifested as a distance change between satellites, which is precisely measured by the two-frequency ranging system. This measurement, in turn, allows the determination of the Earth's gravity field and its temporal variations. To aid in separating the gravitational from nongravitational perturbations, a SuperStar accelerometer, developed by ONERA, measures the surface forces acting on the spacecraft. A Black Jack Global Positioning System receiver developed by JPL provides additional position and time information, as well as the onboard data processing.

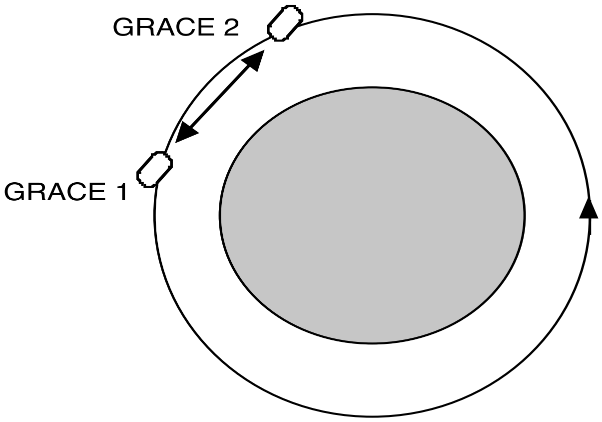

Figure 6. Configuration of GRACE satellites.

GRACE was designed to help unravel global climatic issues by enabling a better understanding of ocean surface currents and heat transport, measuring changes in sea-floor pressure, watching the mass of the oceans change, and by monitoring changes in the storage of water, snow and ice on the continents. It is expected that GRACE will provide a dramatic improvement over current gravity models such as EGM96; better than two orders of magnitude for some resolution ranges. Preliminary results, shown in Figure 7, demonstrate substantial progress towards this goal. The actual errors in this preliminary solution are probably above the formal errors, but it is expected that the GRACE results will eventually draw closer to the baseline performance curve. Even if GRACE were to perform no better than the current formal errors, the only major source of uncertainty in the static gravity field remaining would be in the dominant J_2 coefficient. This problem is eliminated by using the two node rates to recover the LT parameter and J_2. In addition, the correlations between the low-degree zonals from GRACE are considerably lower than for models such as EGM96. For example, the correlation between J_2 and J_4 is –0.93 for EGM96 (indicating very poor separation) but less than –0.1 for GRACE. The correlation between J_4 and J_6 is –0.80 for EGM96 but only –0.24 for GRACE, and the correlation between J_4 and J_8 is –0.65 for EGM96 but an insignificant -0.02 for GRACE.

Using only the nodes removes problems with the uncertain models for the surface forces that strongly affect the perigee (but have only minor effects on the orbit nodes). GRACE will also provide estimates of the seasonal and long-period gravity variations, and the ocean tide models are increasing in accuracy as the TOPEX/Poseidon, Jason-1 and other altimeter missions continue to collect data. Tide modeling errors which remain can be reduced by averaging over a longer period of time or through the removal of specific periodic terms (as was done in *Ciufolini et al.* [1998]).

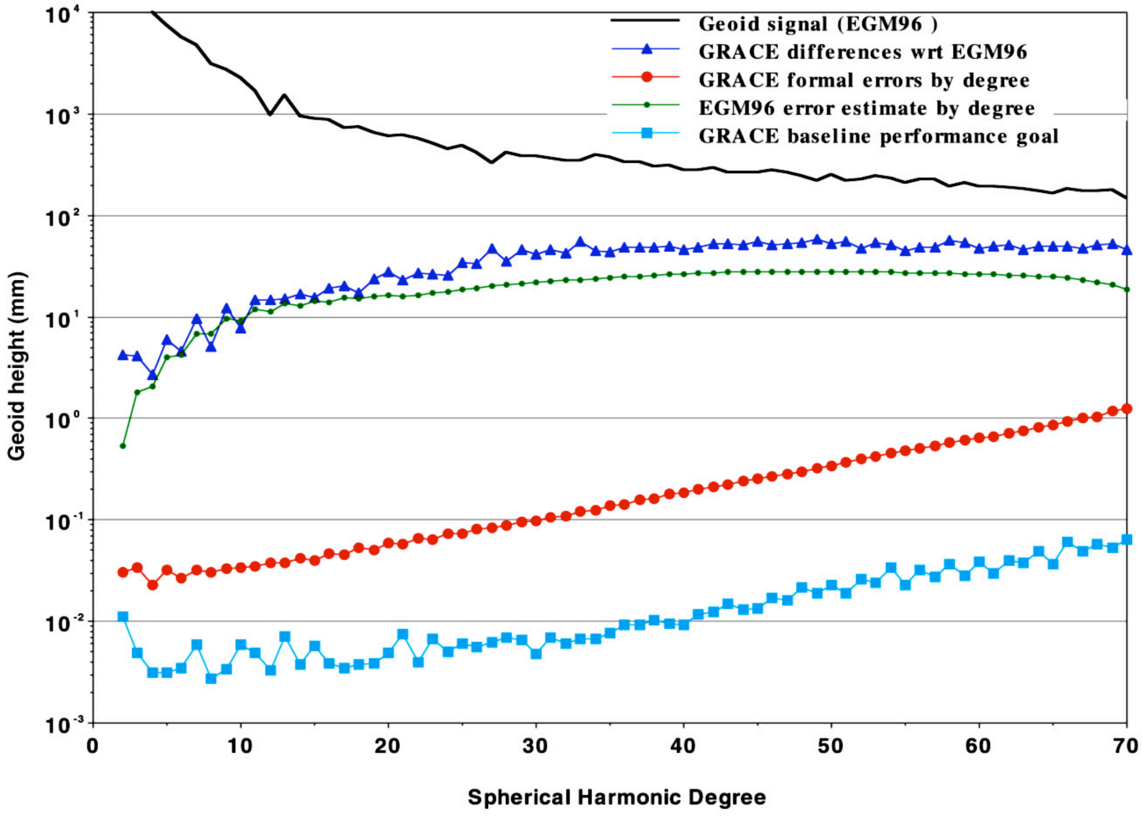

Figure 7. Preliminary gravity field model improvement from GRACE.

Summary

We expect GRACE to dramatically improve our knowledge of the static and temporal components of the gravity field, making a confident Lense-Thirring recovery possible from the existing LAGEOS-1 and LAGEOS-2 satellites. *Peterson* [1997b] examined the possible improvement in the Lense-Thirring experiment assuming a successful GRACE mission and estimated an error of approximately 16%. However, the analysis was pessimistic in some regards, since there was no accommodation of the tidal or seasonal gravity variations, which were the largest contributors to the error. As noted previously, the tide models will continue to improve, a longer time average can be used, specific periodicities in the node variations can be removed, and GRACE should provide accurate estimates of the seasonal and long-period variations in the low-degree zonals. A more current error assessment is probably at the few percent level, although a precise error estimate will have to await the conclusion of the GRACE mission. This should be more than adequate for a confident detection of frame-dragging and test of General Relativity.

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