Lunar Geophysics, Geodesy, and Dynamics

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Abstract

Experience with the dynamics and data analyses for Earth and Moon reveals both similarities and differences. Analysis of Lunar Laser Ranging (LLR) data provides information on the lunar orbit, rotation, solid-body tides, and retroreflector locations. Lunar rotational variations have strong sensitivity to moments of inertia and gravity field while weaker variations, including tidal variations, give sensitivity to the interior structure, physical properties, and energy dissipation. A fluid core of about 20% of the Moon's radius is indicated by the dissipation data. The second-degree Love numbers are detected, most sensitively k_2. Lunar tidal dissipation is strong and its Q has a weak dependence on tidal frequency. Dissipation-caused acceleration in orbital longitude is dominated by tides on Earth with the Moon only contributing about 1%, but lunar tides cause a significant eccentricity rate. The lunar motion is sensitive to orbit and mass parameters. The very low noise of the lunar orbit and rotation also allows sensitive tests of the theory of relativity. Moon-centered coordinates of four retroreflectors are determined. Extending the data span and improving range accuracy will yield improved and new scientific results.

Introduction – the Earth and Moon

This paper gives an overview of lunar geophysics, geodesy, and dynamics. There is a three-decade span of laser ranges to the Moon. The Satellite Laser Ranging (SLR) and Lunar Laser Ranging (LLR) techniques are analogous and there has been parallel evolution of equipment and range accuracy and parallel efforts in model development and data analysis. It is natural to compare and contrast the SLR and LLR modeling and data analysis experiences for the Earth and the Moon. The requirement for accurate Earth rotation and orientation, solid-body tides, and station motion is common to SLR, LLR, GPS, and VLBI analyses. For SLR and LLR there are parallel concerns about orbital dynamics and the influencing forces. For both the Earth and Moon there is a need for accurate rotational dynamics and there are concerns about accurate body-centered site positions including tidal variations. But there are differences in the details and some of these differences are major.

Consider some differences between the Earth and Moon. The Moon has no plate motion. There are moonquakes, but the largest (magnitude ~5) are much smaller than the largest earthquakes [*Goins et al.*, 1981a]. The Moon's rotation is synchronous and slow (27.3 days). The Moon's rotational speed at the equator is 1% of the Earth's speed and, unlike the Earth, the Moon's geometrical [*Zuber et al.*, 1994; *Smith et al.*, 1997] and gravitational figures [*Dickey et al.*, 1994; *Lemoine et al.*, 1997; *Konopliv et al.*, 1998, 2001] are not near rotational equilibrium. The Moon lacks an atmosphere and oceans. Consequently, the rotation is quiet, there is no tidal loading, and there is no analog to geocenter motion. Because of the large lunar mass, nongravitational accelerations on the orbital motion are much smaller than artificial Earth satellites experience. Owing to the tectonic stability and quiet orbital and rotational dynamics, the full three-decade span of LLR data can be fit with a single unbroken orbit. Ranges from recent years have a post-fit rms residual of 17 mm.

Less is known about the Moon's interior structure than the Earth's. There is scant information about the structure below the deepest moonquakes at ~1100 km depth. It is only recently that moment of inertia [*Konopliv et al.*, 1998] and magnetic induction evidence [*Hood et al.*, 1999] for a lunar core have firmed up and that LLR analyses have indicated that the lunar core is liquid [*Williams et al.*, 2001]. It is still unknown whether there is a solid inner core within the liquid. Such unknowns complicate the data analysis, but they are opportunities for scientific discovery.

Science from the Lunar Orbit

The orbits of familiar artificial Earth satellites can be visualized as precessing, inclined ellipses perturbed most strongly by the Earth's J_2 and less strongly by other irregularities in the gravity field as well as by external bodies such as the Moon and Sun. For the lunar orbit (Table 1) the Sun is a very strong perturber and the nonspherical gravity fields of the Earth and Moon are weaker influences. For data analysis numerical integration of the lunar and planetary orbits is used, but series representations of the lunar orbit variations aid understanding. Table 2 presents the largest few periodic terms in lunar radial distance for several classes of gravitational interactions: solar and planetary interactions and gravitational harmonics [*Chapront-Touzé and Chapront*, 1988; 1991; *Chapront-Touzé*, 1983; C_{22} this paper], relativity excluding constant changes in scale [*Lestrade and Chapront-Touzé*, 1982; *Nordtvedt*, 1995; this paper], and solar radiation pressure [*Vokrouhlicky*, 1997]. Despite the great size of the solar and planetary perturbations, they are accurately computed with numerical integration and the ranges are accurately fit to the centimeter accuracy of the LLR data. For example, modern data analyses can determine isolated (in period) orbit terms with a few millimeters accuracy so the large solar perturbation terms correspond to nine digits of accuracy for the mass ratio of Sun/(Earth+Moon). Note that the largest *nongravitational* perturbation of the radial component is 4 mm due to solar radiation pressure [*Vokrouhlicky*, 1997]. Table 3 shows contributions to precession rates from several types of interactions [*Chapront-Touzé and Chapront*, 1983]. Note that nine digits of accuracy for the solar interaction corresponds to <1 mas/yr in precession rates so that even small effects such as lunar gravity and relativity are important. The precession times are 6.0 yr for argument of perigee, 8.85 yr for longitude of perigee, and 18.6 yr for node. The large distance causes the lunar orbit to be virtually unaffected by atmospheric drag and other unpredictable, variable geophysical effects.

Table 1. Lunar orbit general characteristics.

Mean distance	385,000.5 km
a from mean 1/r	384,399.0 km
e	0.0549
i to ecliptic plane	5.145˚
Sidereal period	27.322 d

Table 2. Largest periodic terms in series for lunar radial variations for different classes of interactions.

Interaction Type	**Amplitudes**
Ellipticity	20905 km
Solar perturbations	3699 and 2956 km
Jupiter perturbation	1.06 km
Venus perturbations	0.73, 0.68 and 0.60 km
Earth J_2	0.46 and 0.45 km
Moon J_2 & C_{22}	0.2 m
Earth C_{22}	0.5 mm
Lorentz contraction	0.95 m
Solar potential	6 cm
Time transformation	5 and 5 cm
Other relativity	5 cm
Solar radiation pressure	4 mm

Table 3. Contributions to orbital precession rates for mean longitude of perigee and node due to various interactions [*Chapront-Touzé and Chapront*, 1983].

Interaction Type	$d\varpi/dt$ "/yr	$d\Omega/dt$ "/yr
Solar perturbations	146,425.38	–69,671.67
Planetary perturbations	2.47	–1.44
Earth J_2	6.33	–5.93
Moon J_2 & C_{22}	–0.0176	–0.1705
Relativity	0.0180	0.0190

Table 4. Parameter values for two ephemerides.

	DE403	DE336
AU	149,597,870.691 km	149,597,870.690 km
Mass ratio Sun/(Earth+Moon)	328900.560	328900.559
Mass ratio Earth/Moon	81.300585	81.300564
dn/dt	–25.7 "/cent^2	–25.7 "/ cent^2
Lunar core	solid	liquid

The ephemerides of the Moon and planets are generated by numerical integration. The initial conditions of the motion, mass ratios, and other solution parameters come from joint fits of the lunar and planetary data. Table 4 shows only minor differences for some of the parameter values for two ephemerides, DE403 [*Standish et al.*, 1995] and DE336 (not exported), which were generated several years apart with different lunar core models. Files and documentation for lunar and planetary ephemerides and lunar physical librations are available at the web site http://ssd.jpl.nasa.gov/horizons.html. The DE403 export ephemeris is recommended for the Moon.

Tidal dissipation in the Earth and Moon causes secular changes in the lunar semimajor axis a, sidereal mean motion n, orbital period, eccentricity e, and inclination i. Based on three ephemerides with iterated fits and integrations (DE403, DE330, and DE336) the total sidereal dn/dt is –25.7 "/cent^2, which is equivalent to 37.9 mm/yr for total da/dt (taking a definition where a=384,399 km). Of the total dn/dt value, dissipation in the Moon accounts for +0.3 "/cent^2 [*Williams et al.*, 2001] and dissipation in the Earth causes –26.0 "/cent^2. The diurnal and semidiurnal contributions are not known as well as the total but they are close to –4 and –22 "/cent^2, respectively. Dissipation in the Moon is mainly determined from lunar rotation rather than orbit. The three ephemeris values for the total dn/dt only spread by 0.03 "/cent^2 while the uncertainty in the theory for converting LLR model parameters (Love numbers and time delays) to dn/dt and da/dt is thought to be several times that. The eccentricity rate presents a mystery. Lunar tidal dissipation has a significant influence on this rate and the total rate should be the sum of Earth ($1.3\text{x}10^{-11}$ /yr) and Moon ($-0.6\text{x}10^{-11}$ /yr) effects. However, we find it necessary to solve for an additional anomalous eccentricity rate of $(1.6\pm0.5)\text{x}10^{-11}$ /yr [*Williams et al.*, 2001] when fitting the ranges. The anomalous rate is equivalent to an additional 6 mm/yr decrease in perigee distance. Note that the integrated ephemerides (and DE series) do not include the anomalous eccentricity rate.

The great stability of the lunar orbit allows LLR to make accurate tests of gravitational physics [*Nordtvedt*, 1996]. LLR tests of the equivalence principle show that the Earth and Moon are attracted alike by the Sun's gravity with

$$\Delta\text{acceleration / acceleration} = (M_G/M_I)_{\text{Earth}} - (M_G/M_I)_{\text{Moon}} = (-0.7\pm1.5)\text{x}10^{-13}$$

[*Anderson and Williams*, 2001]. Different aspects of relativity theory are described with parametrized post-Newtonian (PPN) β and γ parameters and geodetic precession [*Williams et al.*, 1996a]. *Anderson and Williams* derive

$$|\beta-1|\leq 0.0005$$

from the equivalence principle test while γ, β and geodetic precession are tested by *Williams et al.* [2002b]. A test of the preferred-frame parameter α_1 has been performed [*Mueller et al.*, 1996]. All tests are in agreement with Einstein's General Relativity which is used for the numerical integration of ephemerides. No variation of the gravitational constant is discernible,

$$(dG/dt) / G = (0.0\pm1.1)\text{x}10^{-12} /\text{yr}$$

[*Williams et al.*, 2002b]. The uncertainty corresponds to 1.5% of 1/(age of the universe) showing that the size of the solar system does not share the cosmological expansion.

Positions on the Moon – Lunar Geodesy

There are retroreflector arrays at the Apollo 11, 14, and 15 sites plus the Lunokhod 1 and 2 sites (Figure 1). The four reflectors other than Lunokhod 1 are ranged by the operational LLR stations at McDonald Observatory and Observatoire de la Côte d'Azur (OCA). While there were early reports of ranges from the Lunokhod 1 reflector in the Soviet Union and at Pic du Midi and later reports from Simeis, this reflector has never been acquired by the two main stations.

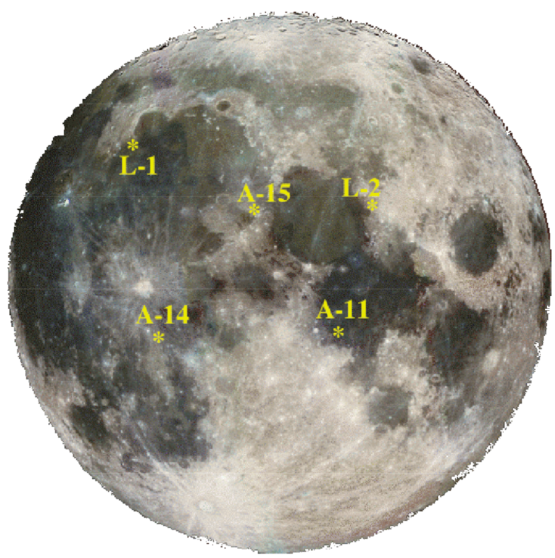

Figure 1. Location of retroreflectors on the Moon.

When determining precise positions on the Earth, SLR and LLR data analysis is aided by a variety of range directions. Since the Moon, on average, keeps one face toward the Earth this variety of directions is lacking for Moon-centered reflector coordinates. As seen from the surface of the Moon the variation in direction of the Earth about the mean, due mainly to the elliptical, inclined, and perturbed orbit, but also to the variations in rotation and orientation, is about 0.1 radian in both the longitude and latitude directions. Consequently, the position accuracies for two Moon-centered retroreflector components (Y and Z) are roughly 0.1 m. The X component, directed towards the mean Earth, acts very much like the mean lunar distance, which depends on orbit scale, which depends on G(M+m), the product of the universal gravitational constant and the sum of the masses of the Earth and the Moon, which depends on the mass ratio Sun/(Earth+Moon). The present uncertainty in the X component and mean lunar distance is 0.4 to 0.6 m, depending on the uncertainty adopted for the relation between lunar tidal Love numbers. The equivalent uncertainty for G(M+m) is 0.0013 to 0.0018 km^3/sec^2 while the uncertainty for the mass ratio Sun/(Earth+Moon) is 0.0011 to 0.0015.

The Earth's coordinate frame is set up so that the z axis is near the principal axis of greatest moment of inertia while the direction of zero longitude is adopted. The synchronous lunar rotation permits all three lunar axes to be defined dynamically. There are two options for the definition: principal axes or axes aligned with the mean Earth and mean rotation directions. The three-dimensional rotation of the Moon, called physical libration, is integrated numerically along with the lunar and planetary orbits. For the integration and fits the lunar coordinate system is aligned with the principal axes since that is a natural frame for expressing the torques and differential equations for rotation, integrating the rotation, and fitting the data. This is the frame of the numerically integrated lunar rotation distributed with DE403 and other modern JPL ephemerides. In the principal axis system the gravitational constants C_{21}, S_{21}, and S_{22} are zero.

Lunar gravitational harmonics which are odd in latitude and odd in longitude, such as S_{31} and S_{33}, cause a small constant rotation of the x and y principal axes about the z axis so that the x principal axis is displaced in longitude from the mean direction to the Earth. Similarly, harmonics which are even in longitude and odd in latitude, e. g., C_{30} and C_{32}, cause a constant rotation of the x and z principal axes about the y axis so that the z principal axis is displaced from the axis of rotation. These rotations depend on the harmonics of the ephemeris or solution. For DE403 the shifts are 63.9" in longitude and 79.1" in pole direction. For use by a lunar geodetic network these shifts in principal axis coordinates are removed and the coordinates are identified as "mean Earth/rotation coordinates". A set of retroreflector coordinates is given by *Williams et al.* [1996b]. The LLR reflector coordinates along with VLBI determined ALSEP transmitter locations serve as control points for the lunar geodetic network [*Davies et al.*, 1987; *Davies et al.*, 1994; *Davies and Colvin*, 2000]. Only three of the Apollo sites (15, 16, and 17) are located on the high quality Apollo mapping camera photographs. The uneven distribution of accurate control points causes the accuracy of the network to vary strongly with lunar position. Future missions to the Moon could improve the accuracy of the network in two ways: a) future landers should carry retroreflectors, and b) future lunar orbital missions should accurately map all of the Apollo sites plus the two Lunokhod sites plus any new lander locations.

Tides on the Moon

Solid-body tides are raised on the Moon by the gravitational attraction of the Earth. These tides provide an opportunity to sample the bulk elastic response of the Moon through the three second-degree Love numbers. The Earth causes a strong tidal potential at the Moon, but the Moon's small size makes it harder to distort than the Earth and its synchronous rotation keeps the angular variation of direction modest. The net result is that the variable tidal distortion of the Moon by the Earth is similar in size to the tidal distortion of the Earth by the Moon. The lunar tidal variations may be represented as sums of periodic components. The largest vertical tides have amplitudes of ~0.1 m with periods of 27.555 d (anomalistic period) and 27.212 d (mean period of nodal crossings of ecliptic plane). Other tidal components >1 cm occur around 1 month and 1/2 month while tidal components >1 mm include periods of 1/3 month, 7 months, 1 yr and 6 yr. Horizontal tides are about half the size of the vertical tides. Sun-raised tides are ~2 mm in amplitude. Both third-degree tides and the analog of the pole tide are less than 1 mm. Due to the synchronous rotation the lunar counterpart to the Earth's familiar M2 tide is unvarying and the varying tidal components arise from changes in the Earth's distance and direction.

LLR detects the tidal displacements of the retroreflectors which are characterized by the lunar Love numbers h_2, for vertical, and l_2, for horizontal. But an order-of-magnitude stronger sensitivity to tides comes through the rotation. The potential Love number k_2 is sensed through the rotation because tidal variations affect the torques and moments of inertia. An LLR determination of this Love number gives

$$k_2 = 0.0257 \pm 0.0025$$

as reported by *Williams et al.* [2003]. There is a concordant determination of $k_2 = 0.026 \pm 0.003$ based on gravity field variations detected by spacecraft orbiting the Moon [*Konopliv et al.*, 2001].

The Love numbers depend on the radial distribution of the elastic properties, most strongly on the rigidity, or shear modulus, or the related S-wave speed. By themselves the second-degree Love numbers cannot determine radial profiles of lunar elastic parameters, but there is additional information available. Analyses of the Apollo seismic events [*Goins et al.*, 1981b; *Nakamura*, 1983; *Khan et al.*, 2000; *Khan and Mosegaard*, 2002] determine the elastic properties of the upper layers of the Moon well, give less certainty for middle zones, and provide little on the central region. When the seismically determined elastic properties are extrapolated from the bottom of the middle zone down to the core of the Moon, the computed model Love number k_2 comes out smaller than the two preceding determinations by an amount comparable to their uncertainties. There is better agreement between model values and measurements if there is a soft zone, such as a partial melt, just above the core [*Williams et al.*, 2002a]. A deep partial melt has previously been proposed to explain high attenuation of seismic S waves in the deepest regions [*Nakamura et al.*, 1973; 1974]. At their present accuracies the Love number determinations are interesting but the uncertainties are too large to determine the amount of any partial melt. Future improvements in the k_2 accuracy should help constrain the elastic properties of the interior, particularly the deeper zones including the core.

The geometrical distribution of retroreflector sites on the Moon determines the uncertainty of the tidal displacement Love number (h_2 and l_2) determinations. Particularly important is the spread of X coordinates, which in decreasing order are 1653, 1592, and 1555 km for the Apollo 14, 11, and 15 arrays, respectively, a variation of only 6%. The Lunokhod 2 reflector with X = 1339 km improves the spread considerably, but this array makes up less than 3% of the data set owing to its small size and sensitivity of returned signal strength to thermal distortion. More Lunokhod 2 data would benefit the determination of the displacement Love numbers. The Lunokhod 1 site with X = 1115 km would make a tantalizing target for tide work.

There is scatter of ~9 km in the reported locations of the lander (Luna 17) and Lunokhod 1. A few laser observations were reported early in the mobile Lunokhod's journey and later after it stopped moving, but those observations have never been publicly distributed or processed with a data analysis program of modern accuracy. It is unclear whether the effectiveness of the Lunokhod 1 array has been compromised or whether position uncertainty and weak signal conspired to prevent its acquisition by standard ranging stations, but a modern search is in order. There are several possibilities: a) a search can be done from Earth using conventional laser ranging; b) high resolution imaging by orbiting spacecraft could locate the Lunokhod rover and Luna lander on the lunar surface; c) a spacecraft passing through the narrow reflected beam of sunlight would see a bright spot if the resolution is better than ~100 m; or d) an orbiting laser altimeter might hit the retroreflector.

Lunar Rotation – Resonances Make a Difference

For both the Earth and Moon the description of the three-dimensional rotation requires three Euler angles which change with time. Two angles describe the orientation of the pole and the orthogonal equator plane, while the third angle gives the rotation about the pole. For the Earth the orientation of the pole requires precession, nutation, and polar motion, while UT1 and effects of equinox motion are needed for the third angle. For the Moon the angular variations are called physical librations. The LLR analysis uses numerical integration for the three Euler angles and most of their partial derivatives, but, analogous to the orbit, series representations offer insight and will be discussed in this section.

Lunar rotation variations cause shifts in lunar ranges which depend on the retroreflector location. For example, a small positive angular rotation about the polar axis causes the Apollo 11 reflector to shift away from the Earth while the Apollo 14 reflector gets closer (Figure 1). The geographical spread of the retroreflectors is important to the rotation determination. At present, lunar rotation terms which are isolated in frequency are determined in solutions with accuracies of 1-2 mas [*Williams et al.*, 2001] or 8 to 17 mm in horizontal motion.

Several things distinguish the rotation of the Moon from that of the Earth: slow rotation, a less oblate shape but otherwise larger gravitational harmonics, multiple resonance periods, and constrained rotation. A resonance causes synchronous rotation with one principal axis (x) pointing on average towards the Earth. The forced obliquity is so large that the mean lunar equator plane precesses along the ecliptic plane with the same 18.6 yr period that the inclined orbit plane precesses along the ecliptic. In addition, it is emphasized that the rotation of the Moon (and other bodies) is three-dimensional (involves three axes) rather than one-dimensional.

Table 5. Second-degree gravity harmonics and lunar moments of inertia from *Konopliv et al.* [1998]. Permanent tides from k_2 are not included in the values. R = 1738 km.

Parameter	Value
J_2	$(203.428 \pm 0.09) \times 10^{-6}$
C_{22}	$(22.395 \pm 0.015) \times 10^{-6}$
(C–A)/B	$(631.486 \pm 0.09) \times 10^{-6}$
(B–A)/C	$(227.871 \pm 0.03) \times 10^{-6}$
$C/m R^2$	0.3932 ± 0.0002
I/mR^2	0.3931 ± 0.0002

The principal moments of inertia about the x (mean Earth), y, and z (polar) axes are A, B, and C, respectively, with A<B<C. The second-degree moments and gravity harmonics are related: $C_{22}=(B–A)/4mR^2$ and $J_2=[C–(A+B)/2]/mR^2$, where m is mass and the adopted R is 1738 km. *Konopliv et al.* [1998] used LLR determinations of moment combinations and Moon-orbiting spacecraft determinations for the two harmonics (Table 5) to obtain the normalized mean moment $I/mR^2 = 0.3931 \pm 0.0002$ and normalized polar moment $C/mR^2 = 0.3932 \pm 0.0002$. Since the normalized moment of a homogeneous sphere is 2/5, the Moon can have only a small dense core. The Earth with its large dense core has $C/mR^2 = 0.3307$.

For a first approximation it is convenient to think of the Moon as a triaxial spheroid with the gravitational attraction from external bodies causing torques on the nonspherical shape. If the lunar orientation is displaced from equilibrium there are three modes of oscillation about the stable equilibrium orientation. These free librations have three distinct periods which depend on the moment-difference combinations of Table 5. The spectrum of lunar orbit variations exhibits many different periods and the torques from the Earth's attraction are modulated in strength and direction by these variations. The three-dimensional dynamical response of the lunar orientation to the torques about the three axes causes the forced physical librations. The periods of the forced librations are set by the orbit variations, but their amplitudes depend on both the strength of the torque components and how close the forcing periods are to the resonant periods, the three free libration periods.

The series representations of Tables 6 and 7 offer insight into the three-axis rotation. Longitude libration is a rotation about the z axis and Table 6 shows that the largest forced longitude libration terms have diverse periods. The free libration period for a small longitude displacement from equilibrium is 2.89 yr. The 3.0 yr term arises from a small torque which is amplified by the near resonant period. The resonance also favors terms at longer periods and reduces the importance of shorter-period effects. At the equator the annual term amounts to 764 m amplitude (1" = 8.4 m for R=1738 km). The 273 yr term arises from Venus perturbations of the lunar orbit and the 6 yr term arises from third-degree harmonics. By comparison, the semidiurnal variation in the Earth's rotation due to its C_{22} and S_{22} harmonics is 0.03 mas or 1 mm! The constant term is the offset of the principal axis from the mean Earth direction (see Section “Positions on the Moon”). The Moon’s synchronous rotation and the associated resonance make the rotational dynamics profoundly different from that of a freely rotating body such as the Earth.

Table 6. Terms for physical libration in longitude.

Period	Amplitude "
1.0 yr	90.7
∞	63.9
3.0 yr	16.8
27.555 d	16.8
273 yr	14.2
206 d	9.9
18.6 yr	7.8
6.0 yr	6.8

The remaining orientation of the Moon is described by the orientation of the equatorial plane or equivalently the direction of the pole normal to the equator. The analog is the Earth’s precession, nutation, and polar motion. The lunar equator and orbit planes precess together and are aligned such that they tilt in opposite directions to the ecliptic plane: 1.54˚ for the former and 5.15˚ for the latter. Thus, the lunar equator is tilted 6.69˚ to its orbit plane. The apparent similarity of the lunar precession to the Earth’s precession is deceptive. The former is a forced precession, the 18.6 yr period matches that of the orbital plane precession, while the Earth’s 26,000 yr precession is a free precession. The free motion for the Moon is an 81 yr retrograde circulation of the pole about the direction in space traced out by forced motion.

How does the series of forced orientation terms compare with the Earth’s nutation? The Earth’s largest nutation term, a 6.9"x9.2" elliptical term (213x285 m) with an 18.6 yr period, is the counterpart to the forced tilt of I=1.54˚! In addition to the precession, there are periodic forced motions of the lunar pole which in Table 7 are referred to a precessing frame. The equator and pole orientations are two-dimensional and the periods depend on whether the reference frame is rotating. In a frame rotating with the Moon two of the terms combine to give a retrograde elliptical motion of 76"x125" (637x1051 m) with a 6.0 yr period. This slow term would act rather like the Earth’s polar motion, but the Earth’s few meter annual and 14 month polar motion components are caused by geophysical effects rather than external torques. The 79" term is the previously discussed offset between the principal z axis and the mean axis of rotation.

Table 7. Physical libration terms for lunar equator and pole orientation. For a coordinate system precessing along the ecliptic plane with the mean orbital node the terms for the equator’s tilt and node angle (times I=0.02692) are tabulated. The first term follows the mean precession.

Period	Tilt amplitude "	I x node amplitude "
∞	5553.6	0.3
27.555 d	99.0	101.3
27.212 d	78.9	78.9
26.878 d	24.6	24.6
18.6 yr	11.8	11.8
13.606 d	10.5	10.1

For both the Earth and the Moon the description of the two-dimensional orientation of the pole and equator requires two Euler angles. For the Earth the two angles come from the precession and nutation series, and this description must be modified by the polar motion, while for the Moon the two angles are computed from numerical integrations or a series such as the one shown in Table 7. For the Earth's precession and nutation the instantaneous spin vector and the pole (z principal axis) are within a few milliarcseconds of one another. The main separation between the spin and pole directions, a few tenths of an arc second equivalent to a few meters, comes from the geophysically-caused seasonal (dominantly annual) and 14 month polar motion components. This simple picture breaks down for the Moon because, unlike the Earth, there are orientation periodicities comparable to or faster than the 27.3 d rotation period. The departure of the spin axis with respect to the pole includes a 74"x124" elliptical 6 yr motion, a monthly 44" linear oscillation, and a 3"x8" counterpart to the free wobble. The spin vector can be displaced more than a kilometer from the principal axis.

The Moon does have an analog to the Earth's 14-month Chandler wobble. There is a lunar free wobble of 3.3"x8.2" (28x69 m) with a 75 yr period in the rotating frame [*Newhall and Williams*, 1997]. In longitude there is a 1.8" libration at the 2.9 yr period. These two modes were first detected by *Calame* [1977]. The longitude libration turns out to be a blend of free and forced terms. After accounting for the forced contribution the free term is about 1.4" in size. The third mode, the free precession, has been detected at 0.02" [*Newhall and Williams*, 1997; *Chapront and Chapront-Touzé*, 1997]. The computed 1/e damping times for the free librations [*Williams et al.*, 2001] are $3x10^4$ yr (longitude), $2x10^5$ yr (free precession), and $2x10^6$ yr (wobble), so the free librations cannot be primordial and a stimulating mechanism is needed. For the longitude libration *Newhall and Williams* find that *Eckhardt's* [1993] suggestion of stimulation by resonance passage is valid. For the wobble mode *Yoder* [1981] has suggested that stimulation occurs from eddies at the fluid-core/solid-mantle interface. Impacts are not a likely cause [*Peale*, 1975; 1976].

What is currently determined by monitoring the lunar rotation? The two moment difference combinations given in Table 5, seven third-degree harmonics [*Dickey et al.*, 1994], Love number k_2, tidal Qs for several periods, a parameter for dissipative coupling between fluid core and solid mantle, and the amplitude and phase of the three free librations are determined.

Lunar rotational dissipative effects have been considered by *Williams et al.* [2001]. LLR detects four dissipation terms and infers a weak dependence of tidal Q on frequency. The tidal Qs are surprisingly low, 37 at one-month tidal period and 60 at 1 yr. This weak dependence of lunar Q on frequency has a sign opposite that proposed for the Earth. Tidal Q is a bulk property which depends on the radial distribution of the material Qs. LLR cannot distinguish the location of the low-Q material, but at seismic frequencies lower-Q material, suspected of being a partial melt, was found for the zone above the core and below the moonquakes. As discussed for tides, a partial melt would also help explain the k_2 value.

A fluid core does not share the rotation axis of the solid mantle. While the equator of the solid Moon precesses, a fluid core can only weakly mimic this motion. The resulting few cm/sec velocity difference at the core-mantle boundary causes a torque and dissipates energy. The combined dissipation from tides and core causes a 0.26" shift in the pole position equivalent to a 9.8" shift in the intersection of the equator and ecliptic planes. The four detected dissipation terms permit the tidal and core effects to be separated [*Williams et al.*, 2001]. Interpreting the core dissipation with Yoder's turbulent boundary layer theory [*Yoder*, 1995] gives 1-σ upper limits for the core radius of 352 km for molten iron and 374 km for an Fe-FeS eutectic, the mix with the lowest melting point (~1000˚ C). The core pressure is ~50 kbar (5 GPa), comparable to the pressure about 150 km deep in the Earth.

A fluid core also exerts torques if the core-mantle boundary (CMB) is oblate. The most recent attempt at detecting flattening effects is encouraging with a solution value somewhat larger than the uncertainty [*Williams et al.*, 2003]. Core ellipticity influences solutions for the Love number k_2 and the previously given value of k_2= 0.0257±0.0025 used this preliminary core ellipticity effect. When core flattening is set to zero the k_2 values are larger which makes them harder to match with model calculations. Core oblateness allows a free precession of the core, free core nutation (FCN). For the Earth the FCN period is 14 months. Using a lunar core oblateness like the total dynamical oblateness *Petrova and Gusev* [2001] have calculated a 145 yr FCN period. Unlike Earth it cannot be assumed that the fixed lunar figure or the CMB oblateness is near equilibrium with mean spin and tidal forces. So a spread of CMB oblateness values predicts FCN periods from roughly one-half century to several centuries.

The present lunar rotation model involves elastic solid body and fluid core. It will become more complicated in the future. Numerically integrated physical librations (no fluid core) are available with DE403 at the JPL ephemeris web site (http://ssd.jpl.nasa.gov/horizons.html).

Science Potential – from Centimeter toward Millimeter Range Accuracies

Further lunar ranging with current accuracies will continue to benefit the scientific investigations discussed in this paper. Many of the science parameters are very sensitive to time span. Examples are relativity parameters, plate motion, and lunar interior parameters. The sensitivity to the dG/dt parameter increases as the square of the time span. Tides, rotation, and interior studies depend on ranges to multiple retroreflectors so more ranges to the smaller reflectors including Lunokhod 2 are desired. Recovery of the Lunokhod 1 array would help further. The determination of lunar science information would be enhanced if future lunar missions carried additional retroreflectors to diverse locations on the lunar surface. The lunar control network would benefit if future high-resolution orbital imaging locates lunar landers. To this wish list we can add a desire for ranging stations at diverse Earth latitudes.

Millimeter accuracy lunar ranging is practical [*Murphy et al.*, 2001] and the orbit should be stable at that level. What scientific benefits would it give? A quick answer is that all of the present results would improve by an order of magnitude, but there are different time scales for different solution parameters. Some results are rapid, for example UT1 can be determined daily. An improved equivalence principle test would require a little more than a year since the test's synodic period (29.531 d) circulates with respect to the anomalistic period (27.555 d) in 412 d. For good separation during solutions some effects require precession time scales. The 6.0 yr argument of perigee period is important for lunar science and orientation. The 18.6 yr node period is important for the mutual orientation of the orbit, ecliptic, and Earth equator planes. Free libration periods and relevant beat periods are decades.

Tests of relativity theory offer exciting possibilities with millimeter data. Einstein's theory of General Relativity has withstood all tests so far. But Einstein's theory is not a quantum theory and it is expected to break down at some small level. Extending the tests of the equivalence principle and PPN parameters by an order of magnitude in accuracy would probe for the breakdown.

What new results can be anticipated with improved accuracy? The solar J_2 could be measured. For lunar science it should be possible to detect: fluid-core/solid-mantle boundary oblateness with certainty, additional tidal dissipation terms, possibly a core moment term, possibly a solid inner core effect, and possibly free core nutation. A tantalizing possiblility is the detection of free libration excitation. If there are discrete stimulating events, it may be possible to detect their influence on the wobble.

Last Words

The LLR modeling and data analysis effort has been scientifically rewarding and interesting. There are both similarities and differences between the lunar and terrestrial experiences. The Earth and Moon travel together, but they are not twins.

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