A NEW APPROACH FOR MISSION DESIGNING OF GEODETIC SATELLITES

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INTRODUCTION

- Geodetic applications
  - associated measurements are frequently sampled
- Crossover points
  - the ground track of a satellite intersects itself on the surface of the earth
  - measures at the same geographic location separated in time
  - relevant in satellite Geodesy:
    - ∗ satellite altimetry (oceanography)
      - · refinement of satellite orbits (Cloutier 1983,. . . , Kozel 1995)
    - ∗ calibration of a gravity field model (Klokocnick & Wagner, 1994)
- Ideal situation:
  - Repeat Ground Track Orbits

PROCEDURE OF MISSION DESIGN

- Experiment requirements:
  - technical limitations of the sensors
  - geographic or geodesic aspects
  - constrain the orbital parameters to a subset of limited values
- First order of J_2 design:
  - provides a rough estimate of the nominal solution
- Refinement of the orbital elements:
  - trial and error $\longrightarrow$ good nominal set of orbital elements
    - "good": the satellite does not drift substantially from the RGT
  - fine tuning of semimajor axis and eccentricity
  - manual iterative sequence

GEODETIG MISSIONS

- Minimize altitude variation:
  - small constant value of e
  - frozen argument of the perigee
- Brouwer's equations of motion $\longrightarrow$ oblate Earth
  - First order of J_2 analytic approximation:
    - no secular variation in a, e, and i
    - regression of the node
    - line of apsides: critical inclination
      - $\sin i > 2/\sqrt{5} \quad \Rightarrow$ advances
      - $\sin i < 2/\sqrt{5} \quad \Rightarrow$ regresses

FIRST ORDER OF J_2 DESIGN

- Lagrange equations of motion (secular variation)

$$
\begin{aligned}
\frac{\mathrm{d}a}{\mathrm{d}t} &= 0 \\
\frac{\mathrm{d}e}{\mathrm{d}t} &= 0 \\
\frac{\mathrm{d}i}{\mathrm{d}t} &= 0 \\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= -\frac{3nJ_2}{2a^2(1-e^2)^2}\left(\frac{5}{2}\sin^2 i - 2\right) \\
\frac{\mathrm{d}\Omega}{\mathrm{d}t} &= -\frac{3nJ_2}{2a^2(1-e^2)^2}\cos i \\
\frac{\mathrm{d}M}{\mathrm{d}t} &= n - \frac{3nJ_2}{2a^2(1-e^2)^2}\left(\frac{3}{2}\sin^2 i - 2\right)
\end{aligned}
$$

FROZEN ORBITS

- Also consider J_3 (Dallas, 1970; Cutting et al. 1978)
- Reduced system: $(s_i \equiv \sin i)$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{3\alpha^2\mu^{\frac{1}{2}}(4-5s_i^2)}{4a^{\frac{7}{2}}(1-e^2)^2}\left[J_2 + \frac{J_3\alpha}{2a}\sin\omega\,\frac{s_i^2 - e\,(1-s_i^2)}{e\,(1-e^2)^2 s_i}\right]$$

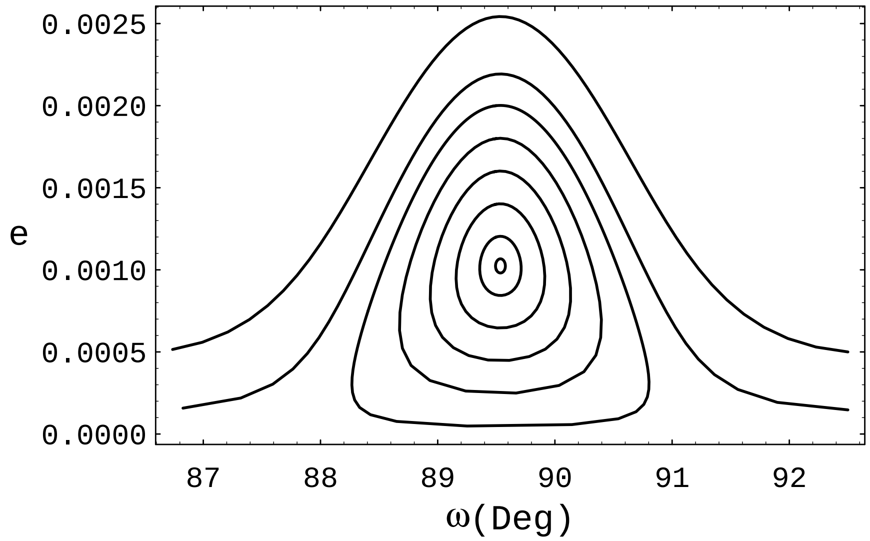
$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{3\alpha^2\mu^{\frac{1}{2}}(4-5s_i^2)}{4a^{\frac{7}{2}}(1-e^2)^2}\,\frac{J_3\alpha}{2a}\,s_i\cos\omega$$

  - critical inclination
    - $\sin i = 2/\sqrt{5} \quad \Rightarrow \quad (\mathrm{d}e/\mathrm{d}t) = (\mathrm{d}\omega/\mathrm{d}t) = 0$
  - low eccentricity:
    - $\cos\omega = 0 \quad \Rightarrow \quad (\mathrm{d}e/\mathrm{d}t) = 0$
    - $s_i^2 - e\,(1-s_i^2) = \gamma\, e\,(1-e^2)^2 s_i \quad \Rightarrow \quad e_0 \approx s_i/\gamma \quad \Rightarrow \quad (\mathrm{d}\omega/\mathrm{d}t) = 0$

$$\gamma \equiv -2(a/\alpha)\,(J_2/J_3) >> 0$$

LIBRATION OF THE PERIGEE

- Interesting property:
  - $e \approx e_0 \quad \Rightarrow \quad$ perigee librates



$$a = 7200.548\,\text{km}, \quad i = 98.7230°$$

(Spot case)

EQUILIBRIA SOLUTIONS

- Frozen orbits as previously defined are NOT equilibria:
  - $(\mathrm{d}a/\mathrm{d}t) \neq 0$, $(\mathrm{d}i/\mathrm{d}t) \neq 0$, etc
  - orbital elements need further adjustment

- More strict definition of frozen orbits (Coffey, Deprit & Deprit, 1994)
  - equilibria solutions of an averaged form of the zonal problem

$$W = \frac{\mu}{r} - \frac{\mu}{r} \sum_{m \geq 2} \left(\frac{\alpha}{r}\right)^m J_m P_m(z/r),$$

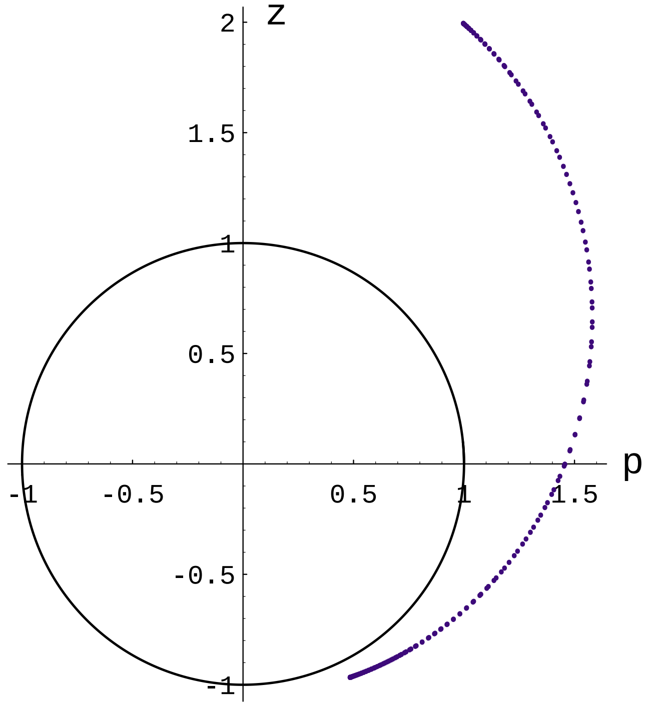
    * average in the mean anomaly and in the node
    * not restricted to a first order averaging
    * nor also limited to J_2 and J_3
      · $J_3 = \mathcal{O}(J_2^2) \sim J_4$ etc.

OUR ALTERNATIVE

Frozen orbits are periodic solutions
of the non-averaged (reduced) problem

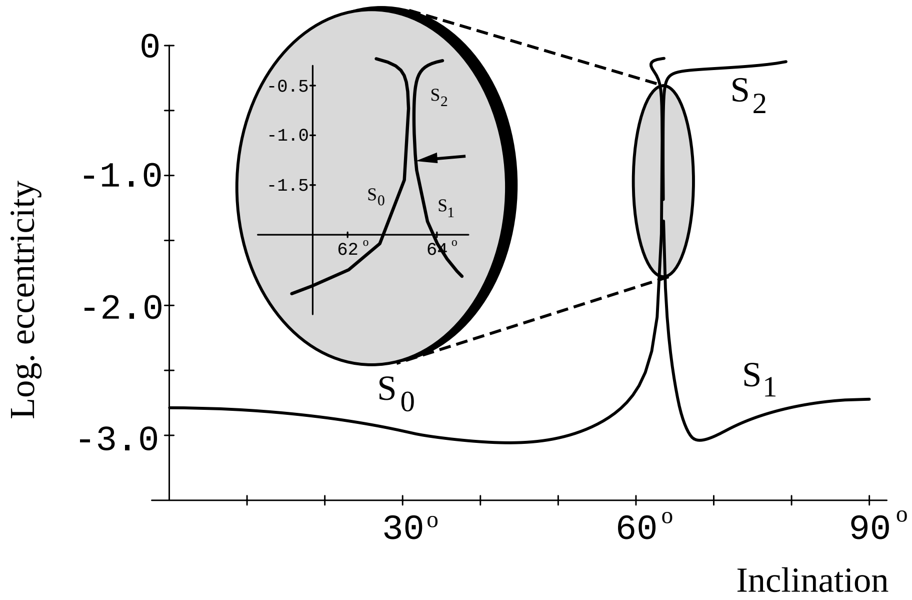
- Zonal problem is axial symmetric
- Cylindrical coordinates $(\rho, \lambda, z, P, \Lambda, Z)$ decouple the problem
  - motion IN the ρ-z plane: 2-DOF
  - motion OF the ρ-z plane:
    - $\lambda = \lambda_0 + \Lambda \int \rho(t)^{-2} dt$
  - polar orbits also periodic in 3-D ($\Lambda = 0 \Rightarrow \lambda = \lambda_0$)
  - otherwise: motion of the node
    - $\rho(T) = \rho_0$
    - $z(T) = z_0$
    - $\dot{\Omega} = (\lambda(T) - \lambda_0)/T$

EXAMPLE: 2-D PERIODIC SOLUTIONS



$a = 10559.26$ km, $e = 0.3463$, $i = 116.556°$, $g = 270°$
(Ellipso Borealis-type)

FAMILIES OF (FROZEN) 2D-PERIODIC ORBITS



Lara, Deprit & Elipe, Celest. Mech. 1995 (9 zonal harmonics)

- Numerical integration
- 2-D differential corrections algorithm
- Poincaré continuation method: z-component of the angular momentum

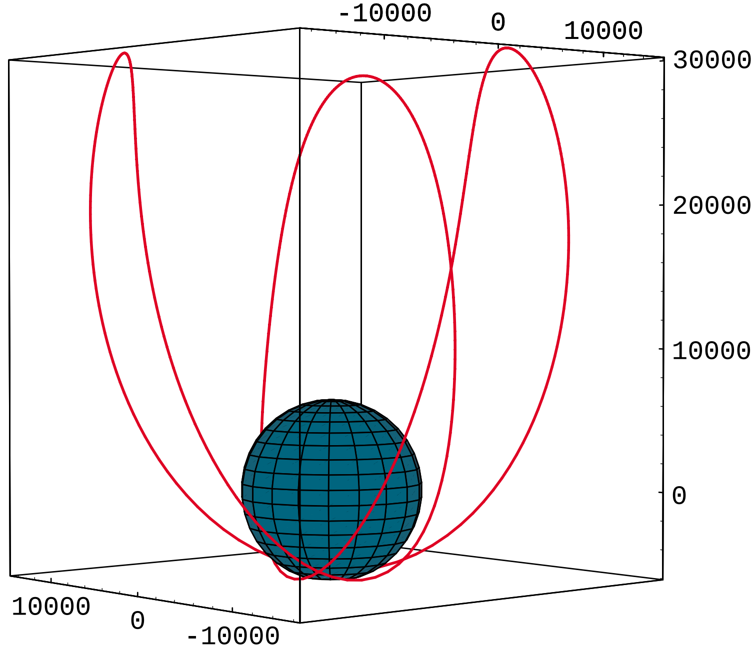
REPEATING GROUND-TRACK ORBITS

- Ground track:
  - path followed by the subsatellite points on the surface of the Earth
- Repeat ground track condition:

$$N\,(\dot\theta_E - \dot\Omega)\,T_\nu = 2\pi\,D$$

  - T_ν length of the nodal period (period in the ρ-z plane)
  - $\dot\theta_E$ Earth rotation rate (assumed constant in the z-direction)
  - $\dot\Omega$ motion of the line of nodes (forced by the equatorial bulge)
  - D number of nodal days (repeat cycle)
  - N number of nodal periods (cycle length)
- Earth fixed (rotating) frame

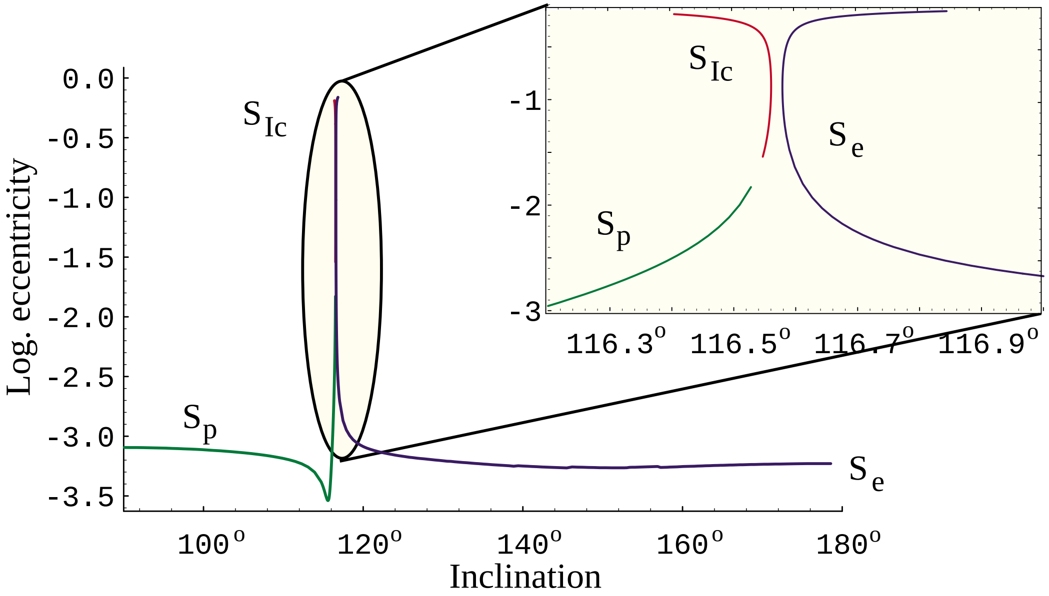
EXAMPLE OF REPEAT GROUND-TRACK ORBIT



3 nodal periods in 1 nodal day

(Molnya-type repeat ground track orbit)

FAMILIES OF REPEAT GROUND-TRACK (3D-PERIODIC) ORBITS

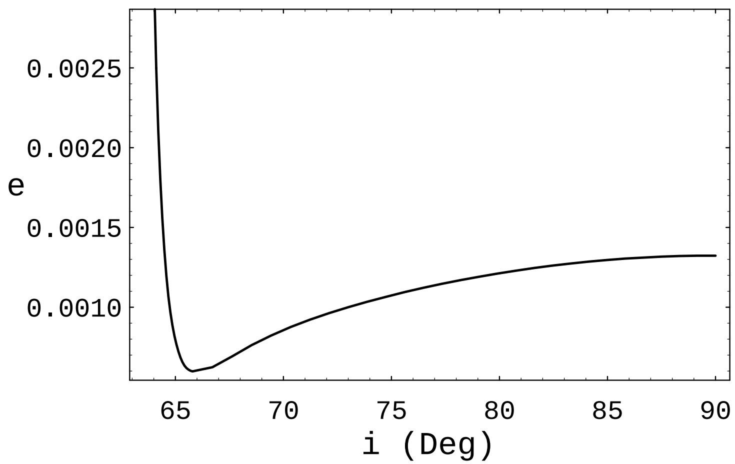


Lara, J. of Astronautical Sciences 1999

- Numerical integration
- 3-D differential corrections algorithm
- Poincaré continuation method. Parameter: Jacobian constant

MISSION DESIGN: GEODETIC SATELLITES

- Jason
  - Repeat cycle 10 days; length 127 periods
  - 2-line orbital elements $i = 66.0444°$, $e = 0.0007776$
- Family S_p of 10/127 RGT 3-D periodic orbits in a rotating frame
  - minimum in eccentricity $i \in [65.86°, 66.13°] \quad \Rightarrow \quad e = 0.0006$



HOW TO FIND (PERIODIC) RGT ORBITS

- Zonal problem is biparametric

$$\mathcal{H} \equiv \frac{1}{2}\left(P^2 + \frac{\Lambda^2}{\rho^2} + Z^2\right) - \frac{\mu}{r} + \frac{\mu}{r}\sum_{m\geq 2}\left(\frac{\alpha}{r}\right)^m J_m P_m(z/r)$$

  - $\mathcal{H}(\rho, z, P, Z, \Lambda) = E$

- Searching in the E-Λ plane we find 2-D periodic orbits that are RGT orbits
  - Repetition cycle related with the semimajor axis $\Rightarrow$ E

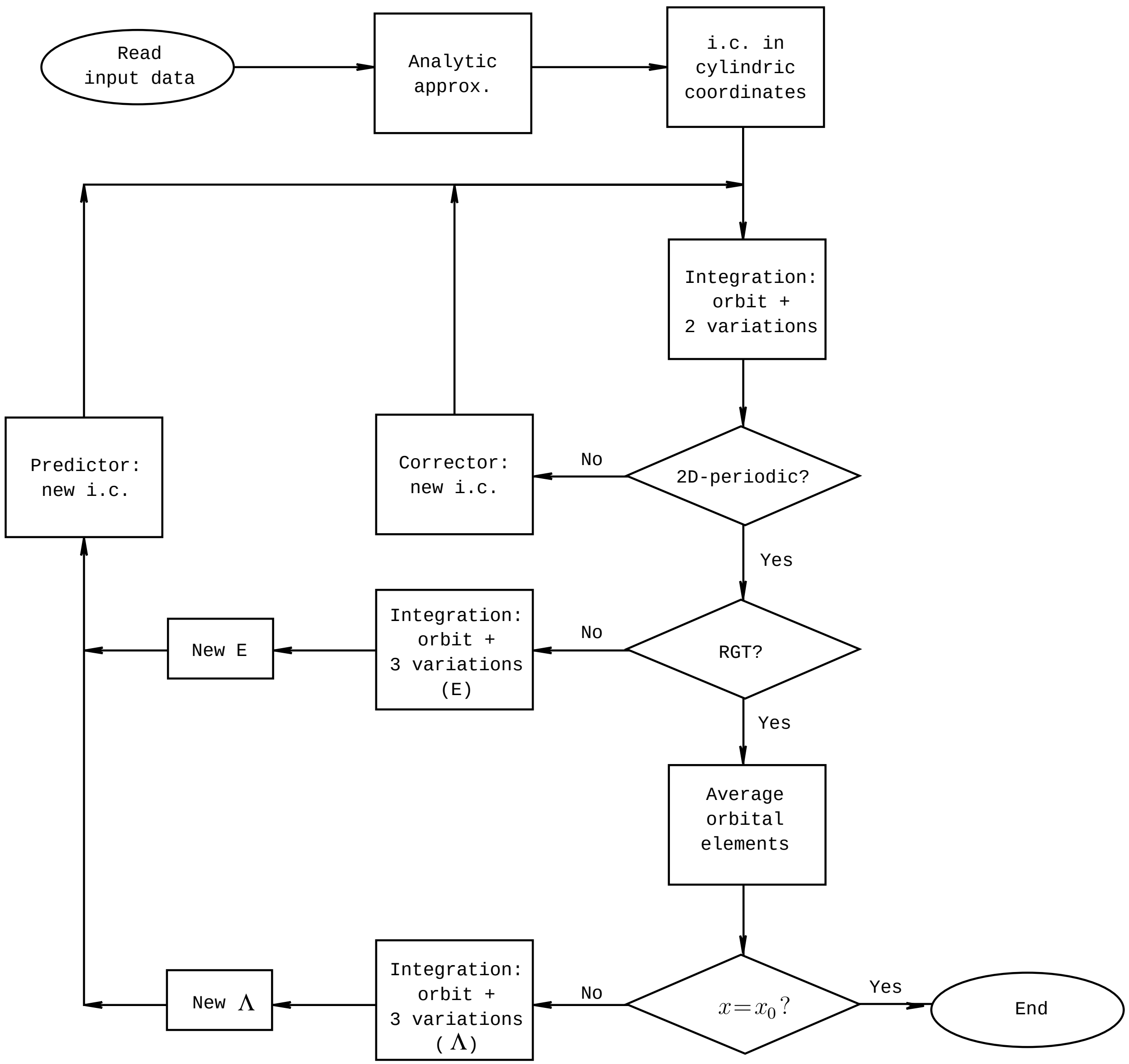
$$\dot{\theta}_E/D = n/N, \qquad n = \sqrt{\mu/a^3},$$

  - Node rate $\dot{\Omega}$ related with the inclination $\Rightarrow$ Λ

$$\dot{\Omega} = \frac{\lambda(T) - \lambda_0}{T} = \frac{\Lambda}{T}\int_0^T \frac{dt}{\rho(t)^2}$$

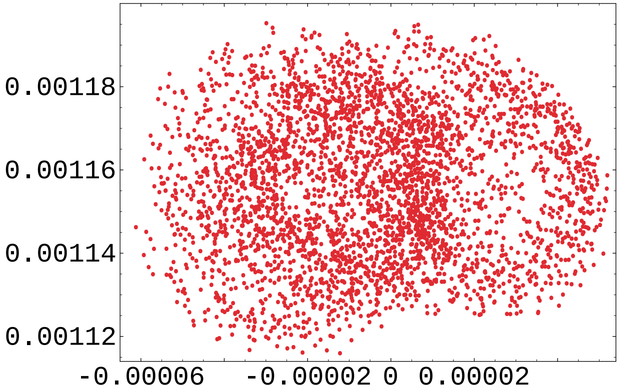
PRACTICAL PROCEDURE

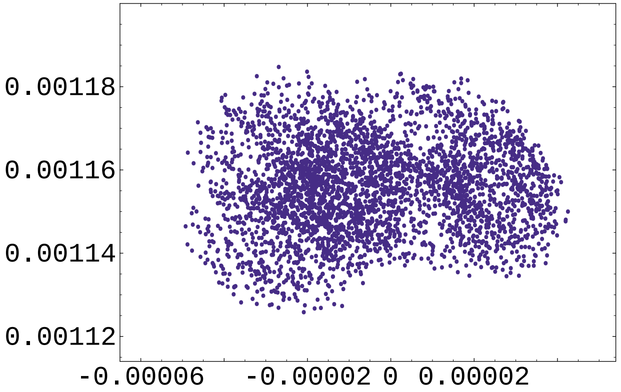
1. First order of J_2 analytic approximation
   - approximate initial conditions in cylindrical coordinates
   - 2-D periodic (frozen) orbit $\approx N/D$ RGT
2. Continuation for variations of E until exactly N/D RGT
3. If necessary, continuation for variations of Λ until
   - sun synchronous
   - desired inclination (almost circular orbits)
   - desired eccentricity (critically inclined orbits)
4. Totally automated: SADSaM
   - a Software Assistant for Designing SAtellite Mission
   - Input: Repeat cycle (N, D), gravitational model, i (or e, or sun synch.)
   - Output: a, e, i, ω, Ω (averaged) —AND— $(x, y, z, \dot{x}, \dot{y}, \dot{z})$


Read
input data
Analytic
approx.
i.c. in
cylindric
coordinates
Integration:
orbit +
2 variations
Predictor:
new i.c.
Corrector:
new i.c.
No
2D-periodic?
Yes
No
RGT?
New E
Integration:
orbit +
3 variations
(E)
Yes
Average
orbital
elements
New Λ
Integration:
orbit +
3 variations
(Λ)
No
$x = x_0$?
Yes
End

ENVISAT

- Sun synchronous; repeat cycle 35 days; length 501 orbits;
  - SADSaM: $a = 7159.49$, $e = 0.00114$, $i = 98.5446°$, $\omega = 90°$
  - Real: $a = 7159.49$, $e = 0.00115$, $i = 98.5425°$, $\omega = 91.9°$
- Propagation 2500 days, GEMT-1 36×36, Moon, Sun (Itziar Barat, ESTEC)





  - Left: real. Right: SADSaM
  - Abscissas $e \cos \omega$, ordinates $e \sin \omega$

CONCLUSIONS

- RGT configurations are highly desirable for missions of geodetic satellites
- RGT orbits are 3D periodic solutions (gravitation only) in a rotating frame
- Classical approach for mission designing is based on trial and error
- Contrary, SADSaM do the job in a totally automated way