Relativity and Earth Orientation Parameters from Lunar Laser Ranging

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Abstract

Lunar Laser Ranging (LLR) is carried out for more than 39 years. Several Newtonian and relativity parameters of the Earth-Moon system can be fitted in a global adjustment with high accuracy. Here, we just give a few examples for gravitational physics parameters, but mainly focus on Earth orientation parameters (EOP). In the global adjustment long-term lunisolar nutation coefficients for different periods (18.6 years, 9 years, 1 year, 182.6 days, 13.6 days) are determined and compared with results from studies of other analysis centres and values of the MHB2000 model (Mathews et al., 2002). Furthermore, the post-fit residuals of the global adjustment are investigated by the daily decomposition method to study variations in $\Delta UT0$ and latitude $\Delta\phi$. In our recent LLR analysis, also different EOP series are applied as input and their effect on the Earth-Moon parameters is investigated.

Model and Analysis

At the Institut für Erdmessung (IfE) the existing model to analyse LLR data is based on Einstein's theory of gravity. It is fully relativistic and complete up to the first post-Newtonian ($1/c^2$) level (Müller et al., 2008). The Barycentric Celestial Reference Frame (BCRF) is the central frame of the analysis. The basic observation equation for the station-reflector distance is defined in this system and the station and reflector coordinates have to be transformed from their respective reference frames (the terrestrial (TRF) or selenocentric (SRF) reference frame) into the inertial frame. In the transformation, the Earth orientation parameters are used for the Earth and the libration angles, computed by numerical integration, for the Moon. The Earth-Moon distance is obtained by numerical integration of the corresponding equation of motion, considering many Newtonian and relativistic contributions.

In our LLR analysis, two groups of parameters for the Earth-Moon system (ca. 180 in total) are determined by a weighted least-squares adjustment of the observations. The first group are the so-called Newtonian parameters, e.g.,

- initial position and velocity of the Moon,
- parameters of physical librations of the Moon,
- coordinates of LLR observatories and retro-reflectors,
- orbit and mass of the Earth-Moon system,
- lunar gravity field,
- long-periodic nutation parameters,
- the lag angle, indicating the lunar tidal acceleration.

Figure 1 shows the annually averaged weighted post-fit residuals of the standard solution with data from Jan. 1970 to Mar. 2008 (16230 normal points). Up to the middle 80ies, the precision of the LLR measurements and analysis model is about 20-30 cm. From 1985 on, more stations started to track the Moon and the residuals decreased. In the last years, only two stations, one with reduced accuracy, observed the Moon, so that

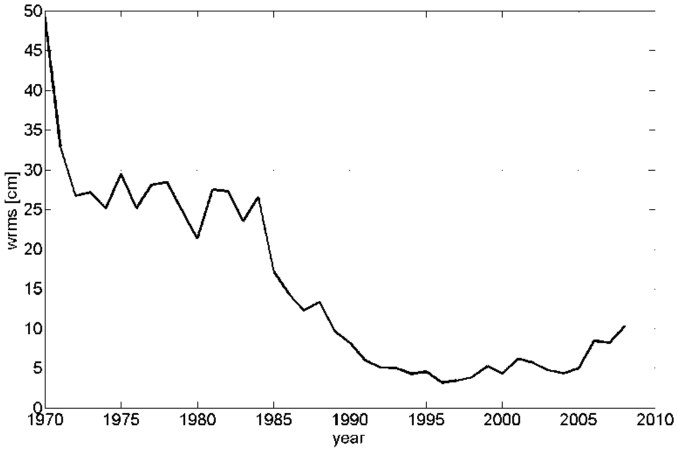

Figure 1. Weighted post-fit residuals (observed minus computed Earth-Moon distance) annually averaged

the residuals increased again. For more details see (Müller et al., 2008). The post-fit residuals of this solution are further investigated w.r.t. Earth orientation parameters, see below.

Relativity

By extending the standard solution, it is possible to solve for parameters related to general relativity, like the temporal variation of the gravitational constant (Müller and Biskupek, 2007), metric parameters as well as the strong equivalence principle and preferred-frame effects (Soffel et al., 2008). The latter can be related to gravito-magnetic effects in the solar system constraining possible deviations from Einstein's theory. (Müller et al., 2008) carried out dedicated sensitivity studies to analyse the potential of LLR to determine specific gravitational physics parameters. Recent results for some selected relativity parameters including realistic errors are given in table 1.

Table 1. Values from LLR for some relativistic parameters and their realistic errors

Nordtvedt parameter η (test of the strong equivalence principle)	$(6 \pm 7)\ 10^{-4}$
time variable gravitational constant $\dot{G}/G$ [yr^{-1}] $\ddot{G}/G$ [yr^{-2}]	$(2 \pm 7)\ 10^{-13}$ $(4 \pm 5)\ 10^{-15}$
preferred frame parameters α_1 α_2 (coupled with velocity of the solar system)	$(-4 \pm 9)\ 10^{-5}$ $(2 \pm 2)\ 10^{-5}$
preferred frame parameter α_1 (coupled with dynamics within the solar system)	$(1.6 \pm 4)\ 10^{-3}$

Nutation

As mentioned in the first section, amplitudes of long-term nutation coefficients can be determined by the analysis of LLR data as part of the global adjustment. The IAU 2000 nutation model is described in the IERS Conventions 2003 (McCarthy and Petit, 2004) as a series for nutation in longitude Δψ and obliquity Δε, referred to the mean ecliptic of date:

$$\Delta\psi = \sum_{i=1}^{N} \left(A_i + A_i^{'} t\right) \sin\left(ARG\right) + \left(A_i^{''} + A_i^{'''} t\right) \cos\left(ARG\right) \tag{1}$$

$$\Delta\varepsilon = \sum_{i=1}^{N} \left(B_i + B_i^{'} t\right) \cos\left(ARG\right) + \left(B_i^{''} + B_i^{'''} t\right) \sin\left(ARG\right) \tag{2}$$

with t in Julian centuries from epoch J2000 and $ARG = \sum_{j}^{5} N_j F_j$, N_j: integers, F_j: Delaunay parameters.

Table 2. Nutation coefficients from MHB2000 model

period	A_i [mas]	B_i [mas]	A_i'' [mas]	B_i'' [mas]
18.6 years	-17206.42	9205.23	3.34	1.54
182.6 days	-1317.09	573.03	-1.37	-0.46
13.6 days	-227.64	97.85	0.28	0.14
9.3 years	207.46	-89.75	-0.07	-0.03
1 year	147.59	7.39	1.18	-0.19

The model is composed of 678 lunisolar and 687 planetary terms with in-phase and out-of-phase components and their time variations. Basis is the REN2000 nutation solution (Souchay et al., 1999) for the rigid Earth, which is convolved to the nutation model MHB2000 for the non-rigid Earth applying the transfer function of (Mathews et al., 2002). The MHB2000 model is relied on the solution of linearised dynamical equations for each forcing frequency, adding contributions from non-linear terms and other effects not included in the linearised equations. The model improved the IAU 1980 nutation theory by the incorporation of mantle anelasticity, ocean tide effects and electromagnetic couplings of the mantle and the solid inner core to the fluid outer core. Table 2 gives the largest terms of the MHB2000 nutation model. There, the non-time-dependent components for the in-phase (A_i, B_i) and out-of-phase (A_i'', B_i'') parts of nutation in longitude and obliquity are shown.

Table 3. Nutation coefficients from our LLR computation

period	A_i [mas]	B_i [mas]	A_i'' [mas]	B_i'' [mas]
18.6 years	-17201.93	9203.41	3.84	3.88
182.6 days	-1316.88	572.98	-3.25	-0.98
13.6 days	-230.54	99.26	0.16	0.31
9.3 years	207.13	-90.75	1.63	-0.21
1 year	146.83	7.86	0.27	-0.58

Analogue to table 2 the same components were fitted in the global adjustment of our LLR analysis. Table 3 gives the preliminary results. The components, which are marked in grey, show the largest differences between the MHB2000 model (Tab. 2) and our LLR analysis (Tab. 3). These are not completely understood yet, but they are also seen in similar way in the analysis of other groups (Williams, 2008) and must be further investigated.

Earth rotation from LLR data

Post-fit residuals from the global adjustment of the LLR analysis can be further investigated to determine corrections for the components of Earth rotation parallel to a change in longitude ($\Delta UT0$) and latitude (variation of latitude $\Delta\phi$). The latter one was also used to iteratively improve the results of the global standard solution.

To determine $\Delta UT0$ and $\Delta\phi$ from LLR data, the post-fit residuals are sorted by station-reflector combinations and merged in daily sets. One set must include the minimum of three station-reflector pairs. Out of 16230 observations, 1179 daily sets for the station OCA in Grasse and 752 daily sets for the station McDonald in Texas were found. These sets are analysed in a least-squares adjustment (daily decomposition method, see (Dickey et al., 1985)) applying the following model:

$$r(t) = r_{\Delta UT0} + r_{\Delta\phi} + r_n , \tag{3}$$

where the post-fit residuals of the global adjustment are assumed to be caused by contributions from universal time

$$r_{\Delta UT0} = 2\,\Delta UT0\; r_E\; \cos\phi \sin H \cos\delta , \tag{4}$$

from variation of latitude

$$r_{\Delta\phi} = 2\,\Delta\phi\; r_E \left(\sin\phi \cos\delta \cos H - \sin\delta \cos\phi\right), \tag{5}$$

and a part containing other effects r_n like systematic ranging errors and model errors. $\Delta UT0$ and $\Delta\phi$ enter the respective equation as

$$\Delta UT0 = \Delta UT1 + \tan\phi \left(x_P \sin\lambda + y_P \cos\lambda\right) \tag{6}$$

and

$$\Delta\phi = x_P\; \cos\lambda - y_P \sin\lambda \tag{7}$$

with the declination δ and hour angle H of the Moon and the latitude ϕ, longitude λ and radius r_E of the Earth. The pole coordinates x_P/y_P and $\Delta UT1$ are also used for the transformation between the celestial and terrestrial systems in our global fit. There in addition, the long-periodic, diurnal and sub-diurnal effects of the ocean are corrected according to the IERS Conventions 2003.

Moreover, the effect of different EOP series, serving as initial values in our LLR analysis, was investigated. The EOP series IERS EOP C04 (Gambis, 2004) and COMB2006 (Gross, 2007) were used as input for the global standard solution. Then, the post-fit residuals were analysed to determine corrections for universal time $\Delta UT0$ and variation of latitude $\Delta\phi$. Both EOP series were obtained from the combination of "operational" EOP series derived from the various space-geodetic techniques VLBI (Very Long Baseline Interferometry), GPS (Global Positioning System), SLR (Satellite Laser Ranging) and LLR as well as optical observations, see (Gambis, 2004; Gross, 2007). Additionally, in the C04 series DORIS

(Doppler Orbitography and Radiopositioning Integrated by Satellite) data were included. Differences between the series are caused by different time spans, filter techniques to combine the data and the treatment of tidal effects. In the C04 series periods from 5 days to 18.6 years are corrected by the model of (Defraigne and Smits, 1999), in the COMB2006 series periods from 5 days to 35 days are corrected using the procedure of (Yoder et al., 1981). Comparison of the two series (C04 minus COMB2006) shows large differences in the period between 1970 and 1982 in all components (x_P, y_P and $\Delta UT1$). From 1982 on, the differences decrease more and more, for polar coordinates it is near zero now. For $\Delta UT1$ the difference is only a few microseconds today, because of the different treatment of the tidal effects.

As an example, figure 2 shows the resulting $\Delta\phi$ for McDonald when using the different EOP series. The daily solutions (dots), calculated with (5), are smoothed by a spline filter. The curves of the two calculations show large differences in the 70ies and middle 80ies, the period where the two EOP series show large differences, too. From the middle 80ies on, when both EOP series are very similar, also the results are very similar here.

In a next step, these spline-interpolated $\Delta\phi$ values were iteratively used as corrections in the global adjustment. Two calculations were made for both EOP series (figure 3 shows the corresponding results): One as reference without the estimated $\Delta\phi$ values (solid lines) and one with the $\Delta\phi$ correction applied (dotted lines). As in the previous figures, it can be seen, that again in the time span from the middle 80ies on, where the EOP series are very similar, also the residuals are very similar. It is furthermore obvious, that the residuals can not really be improved by using the $\Delta\phi$ correction, because the input EOP series are already very accurate. In the time span up to the 80ies, the residuals using the COMB2006 series are smaller than those based on the C04. It seems, that the COMB2006 series fits better to the LLR analysis than the C04 series. But also here, the results of the global adjustment can not really be improved by applying the determined $\Delta\phi$ correction.

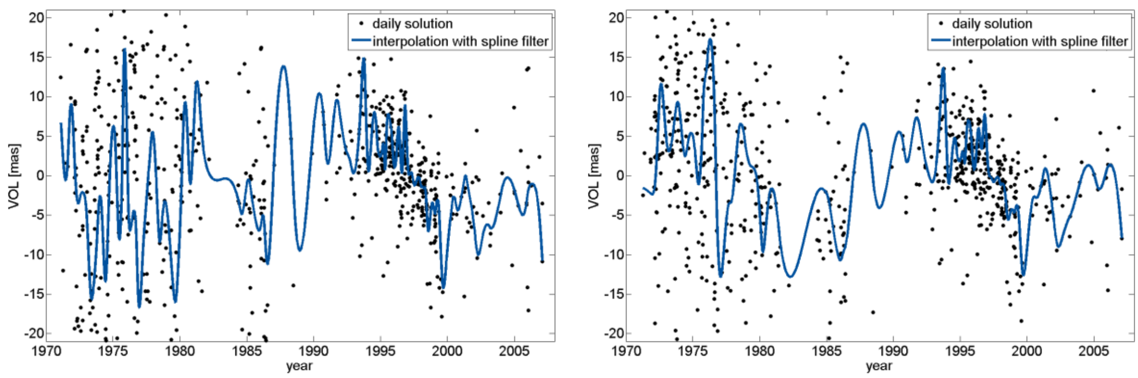

Figure 2. Results for $\Delta\phi$ when using the EOP series C04 (left) and COMB2006 (right) as input in the global adjustment

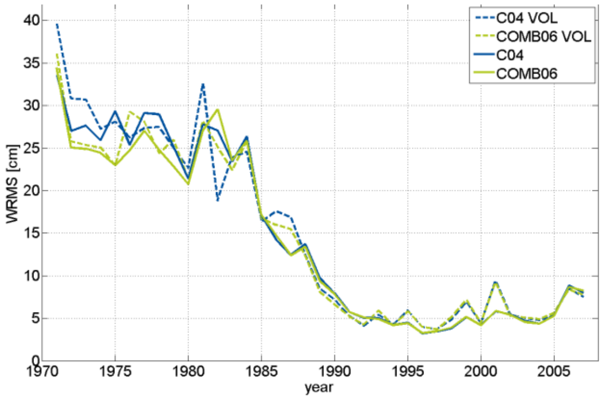

Figure 3. Results of using the Δϕ corrections in the global adjustment

Conclusions

A 38-year LLR data set has been analysed to determine Newtonian as well as gravitational physics parameters with high accuracy, impressively confirming Einstein's theory.

The results of long-periodic nutation coefficients determined from LLR analysis show differences to the MHB2000 model for some of the coefficients. These differences are also seen by other analysis centres, but can not be explained yet and must be further investigated. In a next step, the results from LLR shall be directly compared to VLBI nutation results.

The investigation and determination of the EOP correction parameter Δϕ based on different EOP input series is possible with the daily decomposition method. Using these values in the next iteration step of the LLR analysis, however, does not significantly improve the residuals. Obviously, the input EOP series are already of very good quality, especially from the middle 80ies on. In a future step other filters, apart from the spline filter, will be tested. Also values for length of day *LOD* will be calculated to compare them with *LOD* results from VLBI.

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