Estimation of the elastic Earth parameters k_2 and k_3 from the SLR technique

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Abstract

The global elastic parameters k_2, k_3 associated with the tide variations of the satellite motion are estimated from the Satellite Laser Ranging (SLR) data. The study is based on satellite observations taken by the global network of the ground stations during the period from January 1, 2005 until January 1, 2007 for monthly orbital arcs of satellites Lageos 1 and Lageos 2, separately. The observation equation, contains unknowns which are related to orbital arcs only, some unknowns are common for all arcs and the Earth parameters, can also be estimated. The example is elastic Earth parameters k_2, k_3 which describe tide variations of the satellite motion. The adjusted values k_2 equal to 0.3016 ± 0.0001 and 0.3006 ± 0.0001, k_3 equal to 0.0989 ± 0.0051 and 0.0810 ± 0.0051 for LAGEOS1 and LAGEOS2 tracking data are discussed and compared with geophysical estimations of Love numbers. All computations were performed employing the NASA software GEODYN II (Eddy et al. 1990).

1. Introduction

The attraction of the Earth is the most important force acting upon the artificial satellite. The perturbations produced by the Earth's gravity field evidently belong to the greatest disturbances. Usually, the gravity field and the tide potential (which describe the influence of the conservative forces) in the external point $P(r, \varphi, \lambda)$ of the Earth are given by following expression (1).

$$V = \frac{GM}{r}\left(1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n P_{nm}(\sin\varphi)((C_{nm} + \Delta C_{nm})\cos m\lambda + (S_{nm} + \Delta S_{nm})\sin m\lambda\right) \quad (1)$$

The gravity field is modeled by standard geopotential coefficients C_{nm}, S_{nm} and their tide variations ΔC_{nm} and ΔS_{nm} in time (Eanes et al., 1983) .The tide model (having frequency dependent Love numbers) is computed by a general procedure.

$$\Delta C_{nm} - i\Delta S_{nm} = \frac{k_{nm}}{\sqrt{2n+1}}\sum_{j=2}^{3}\frac{GM_j}{GM_E}\left(\frac{R_E}{r_j}\right)^{n+1} P_{nm}(\sin\Phi_j)e^{-im\lambda_j} \quad (2)$$

where:
$P_{nm}(\sin\varphi)$ - the associated Legendre'a function of degree *n* and order *m*,

GM_j - gravitational parameter for the Moon and Sun,
GM_E - gravitational parameter for Earth.,
R_j - vector from the geocenter to the Moon or Sun,
r, ϕ, λ - vector from geocenter to the station, geocentric latitude and longitude,
k_2 - nominal second degree Love number k_2 equal to 0.3
k_3 - nominal third degree Love number k_3 equal to 0.093 by Longman.

For degree 2 the tide model is computed in two steps. The first step uses a frequency dependent Love number k_2. The changes in normalized second degree geopotential coefficients for step 1 are:

$$\Delta C_{20} = \frac{1}{\sqrt{5}} k_2(0) \frac{R^3{}_E}{GM_E} \frac{GM_j}{r_j^3} P_{20} \sin\varphi_j$$

$$\Delta C_{21} - i\Delta S_{21} = \frac{1}{3}\sqrt{\frac{3}{5}} k_2(1) \frac{R_E^3}{GM_E} \frac{GM_j}{r_j^3} P_{21} \sin\phi_j * \cos\lambda_j \qquad (3)$$

$$\Delta C_{22} - i\Delta S_{22} = \frac{1}{12}\sqrt{\frac{12}{5}} k_2(2) \frac{R_E^3}{GM_E} \frac{GM_j}{r_j^3} P_{22} \sin\varphi_j * \sin 2\lambda_j$$

The above equations are revised in such a way that three different Love numbers $k_2(0), k_2(1), k_2(2)$ for long period, diurnal and semi-diurnal periods are introduced instead of one Love number k_2. If a common nominal number k_2=0.3 is used in equations (2), then the differences between actual and nominal Love numbers must be corrected in step 2 which is described in (Eanes et al., 1983)

The contributions of the degree 3 tides ΔC_{3m} and ΔS_{3m} are computed by a general procedure (2) too. For n=3 m=0 is described by following formula.

$$\Delta C_{30} = \frac{1}{\sqrt{7}} k_3 \frac{R_E^4}{GM_E} \frac{GM_j}{r_j^4} P_{30} \sin\varphi_j \qquad (4)$$

The partial derivative quantities $\frac{\partial\rho}{\partial k_2}$ and $\frac{\partial\rho}{\partial k_3}$ required in expression (5) to estimation of the Love numbers k_2 and k_3 (where ρ is a given SLR measurement) are computed by differentiation of expressions (3,4).
The knowledge of the partial derivatives described above allows to calculate the Love numbers k_2 and k_3 ,using observation equations (5) an iterative process . Initial values used in solution were equal to k_2=0.3 described in IERS Technical Note 21 and k_3 =0.093 (Longman) in (Melchior P, 1978)

$$V_\rho = \sum_{i=1}^{6} \frac{\partial\rho}{\partial\varepsilon}\Delta\varepsilon + \frac{\partial\rho}{\partial k_2}\Delta k_2 + \frac{\partial\rho}{\partial k_3}\Delta k_3 + (O-C) \qquad (5)$$

where:

$\Delta\varepsilon$ - corrections regarded to the satellite arcs,
Δk_2 - correction to the Love number k_2,
Δk_3 - correction to the Shida number k_3,
V_ρ - correction to observation,
$(O-C)$ - observation to satellite minus computed distance between satellite and station.

2. Method of analysis

The study is based on the SLR data of LAGEOS-1 and LAGEOS-2 taken by the global network of SLR stations during the period from 3 January 2005 until 30 December 2006. The database of normal points was processed in 30-day batches. In total 48 orbital arcs were used in the analyses. A preliminary step not reported in this paper, the compression of observations into normal points, was performed for two-minute intervals of LAGEOS-1 and LAGEOS-2 by Crustal Dynamics Data Information System (CDDIS) and EUROLAS Data Center (EDC). Number of normal points used in solution for LAGEOS1 and LAGEOS2 is shown in Fig.1.

The solution was produced employing the software GEODYN II (Eddy et al.,1990). The forces that perturb the satellite orbit needed to be modeled as accurately as possible. The force models used and the reduction of measurements for LAGEOS-1 and LAGEOS-2 are shown in Table 1. The study is based on the station positions published by ITRF2005 referred to the epoch 2000.0.

Table 1. Computation model used in the solution.

DYNAMIC MODEL
Gravity field TEG4 (20,20)
Wahr solid Earth tides NASA/NIMA EGM96 ocean tides
C_R direct solar radiation pressure estimated
Albedo and infrared Earh radiation
Relativistic effects
Accelerations in along-track, cross-track and radial directions (for 5-day intervals)
REFERENCE FRAME
Station coordinates used –ITRF2005 referred to epoch 2000.0
Nutation according to IAU 1980 (Wahr model)
Pole tide
Ocean loading deformation, atmospheric pressure loading deformation
PROCESSING MODE
Normal points provided by CDDIS and EDC
Marini-Murray model for troposphere delay
Center-of-mass correction used
Station dependent data weighted
OBSERVATIONS
LAGEOS-1- 24 arcs normal points (2005.01.03- 2007.01.02)
LAGEOS-2- 24 arcs normal points (2005.01.03- 2007.01.02)
INTEGRATION
Cowell 11-order predictor-corrector; step-size 2 min.

3. Results

The paper presents the global elastic Earth parameters k_2 and k_3 (Love numbers) associated with the tidal variations of satellite orbits estimated for 24 months time interval. The sequential method was adopted for analysis. In the first step, the elastic parameters were adjusted for two orbital arcs. In the next steps, arcs one after the other were included using sequential method. In the each step, the parameters were adjusted once again. The results of analysis are shown in Fig.2 and Fig.3. for k_2 and k_3 separately. The total adjusted value k_2 is equal to 0.3016 ± 0.0001 for LAGEOS1 tracking data and is equal to 0.3006 ± 0.0001 for LAGEOS2 tracking data. The total adjusted value k_3 is equal to 0.0989 ± 0.0051 for LAGEOS1 and 0.0810 ± 0.0051 for LAGEOS2. The discrepancy is at the 0.3% level for k_2 and the 20% level for k_3. The differences from the LAGEOS1 and LAGEOS2 are not significant for k_2 value.

As an example, previous estimation of k_2 by Wahr (1981) provide value equal to 0.299 which have to be corrected.

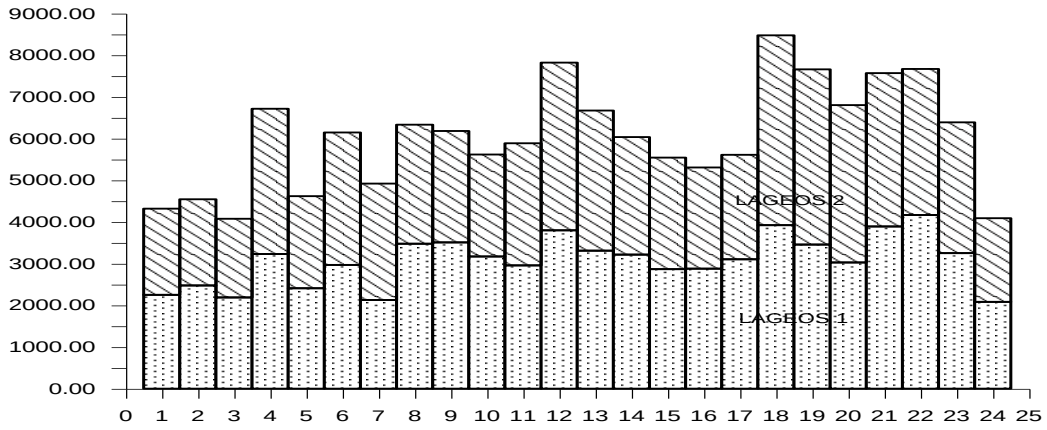

Figure 1. The number of normal points used in solution for LAGEOS1 and LAGEOS2.

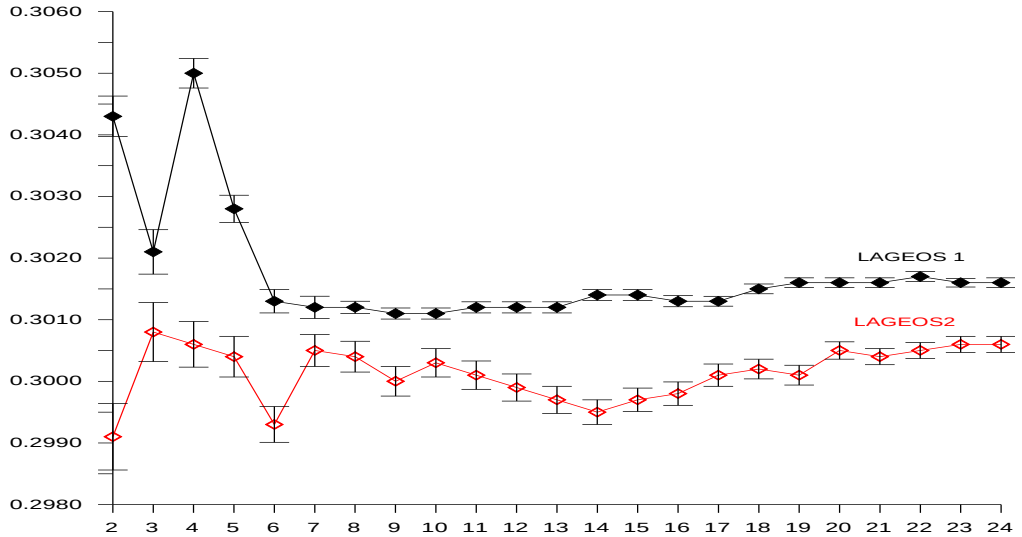

Figure 2. The Love number k_2 estimated from Lageos1 data equal to 0.3016 ± 0.0001 and Lageos2 data equal to 0.3006 ± 0.0001

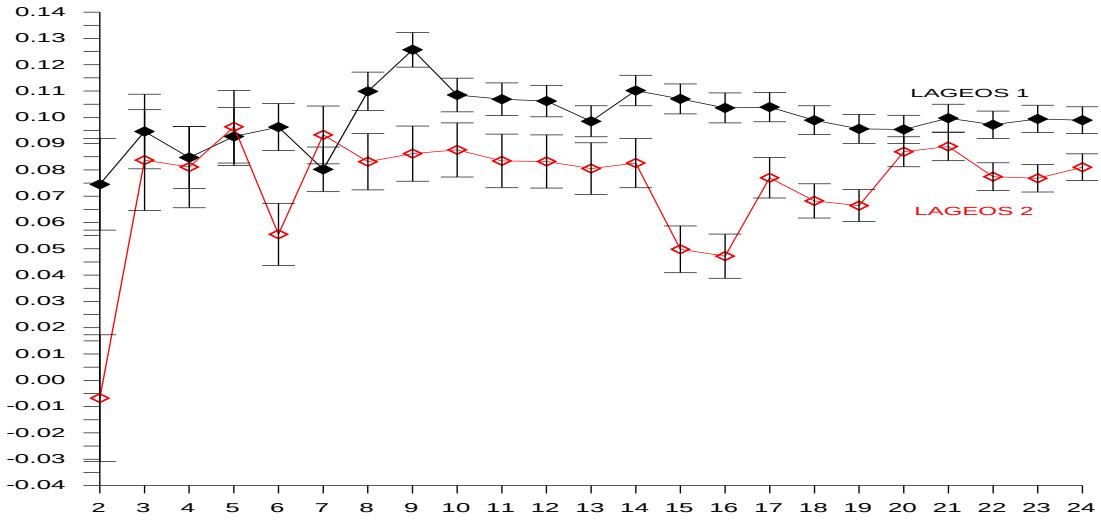

Figure 3. The Love number k_3 estimated from Lageos1 data equal to 0.0989 ± 0.0051 and Lageos2 data equal to 0.0810 ± 0.0051.

4. Conclusions

The SLR tracking data for satellite LAGEOS1 and LAGEOS2 were used to determine the elasticity Earth parameters k_2 and k_3. On the basis of the computations performed it can be concluded that:

- The estimated parameter, Love number k_2 is equal to (0.3016 ± 0.0001 and 0.3006 ± 0.0001) for LAGEOS1 and LAGEOS2 tracking data separately. The good agreement of estimated parameters for both satellites can be seen. Difference is at the level 0.3%.
- The estimated Love number k_3 is equal to (0.0989 ± 0.0051 and 0.0810 ± 0.0051). for LAGEOS1 and LAGEOS2 tracking data separately. Difference is equal to 0.0179, it means at the level 20% value. The discrepancy with value estimated by Longman which is equal to 0.93 (P.Melchior,1978) is agree at the level 5% .
- Stability of estimated elasticity Earth parameter k_2 and their errors became visible for about 23-month time interval (Fig.2). But for k_3, 24-month time interval not allows to obtain stability of solution what is shown in (Fig.3). Probably for number of arc greater than 20 differences are at the level of noise.

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