Validation and estimation of low-degree gravity field coefficients using LAGEOS

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Introduction

Precise orbit determination is an essential task for analyzing SLR data. The quality of the satellite orbits strongly depends on models used for dynamic orbit determination. We discuss the impact of different gravity field models on the LAGEOS-1 and LAGEOS-2 orbits and show that not only the type and maximum degree of the gravity field model is essential, but also the proper choice of a limited number of empirical orbit parameters.

The quality of the estimated orbits is analyzed by analyzing the mean error of the solution based on SLR observations and by comparing predicted orbits with estimated orbits. Orbits resulting from solutions using different gravity field models are directly compared as well.

Eventually, we present first estimates of C_{20} from the two LAGEOS satellites as obtained with the Bernese Software.

Gravity field model	Year	Max. degree	Drift	SLR	CHAMP	GRACE	GOCE	Ground data
JGM3	1994	70		X				X
EGM96	1996	360		X				X
EIGEN-GL04C	2006	360	4	X		X		X
EGM2008	2008	2190				X		X
EIGEN51C	2010	359	4	X	X	X		X
ITG-GRACE2010	2010	180				X		
AIUB-CHAMP03S	2010	100			X			
AIUB-GRACE03S	2011	160	30			X		
GO-CONS-2-DIR-R2	2011	240					X	
GOCO02S	2011	250		X	X	X	X	
AIUB - SST - only	2011	120			X		X	

Tab. 1: List of Earth gravity field models

Models and data

We selected eleven gravity field models for comparison (see Tab.1): Models from the pre-CHAMP era such as JGM3 and EGM96, models including CHAMP data, GRACE data, GOCE data, and combined models.

The 7-day arcs in 2008 were generated using SLR measurements to both LAGEOS satellites according to two different solution strategies: For solutions (a) constant empirical accelerations were estimated per 7-day arc for each satellite in the along-track direction in addition to the initial osculating elements, as well as once-per-rev accelerations in the along-track and cross-track directions. Solutions (b) were based on the same principle, but without estimating the empirical once-per-rev accelerations in the cross-track direction. For both solutions station positions and Earth orientation parameters were co-estimated.

Different gravity field models up to d/o 70 were tested with drifts and degree 1 coefficients taken into account (if available). For each solution the same set of observations was used (data were previously screened using merged normal point data from CDDIS and EDC). 139,000 SLR observations were available in 2008. The number of normal points per week varied between 1932 and 3804 (see Fig. 1a,b).

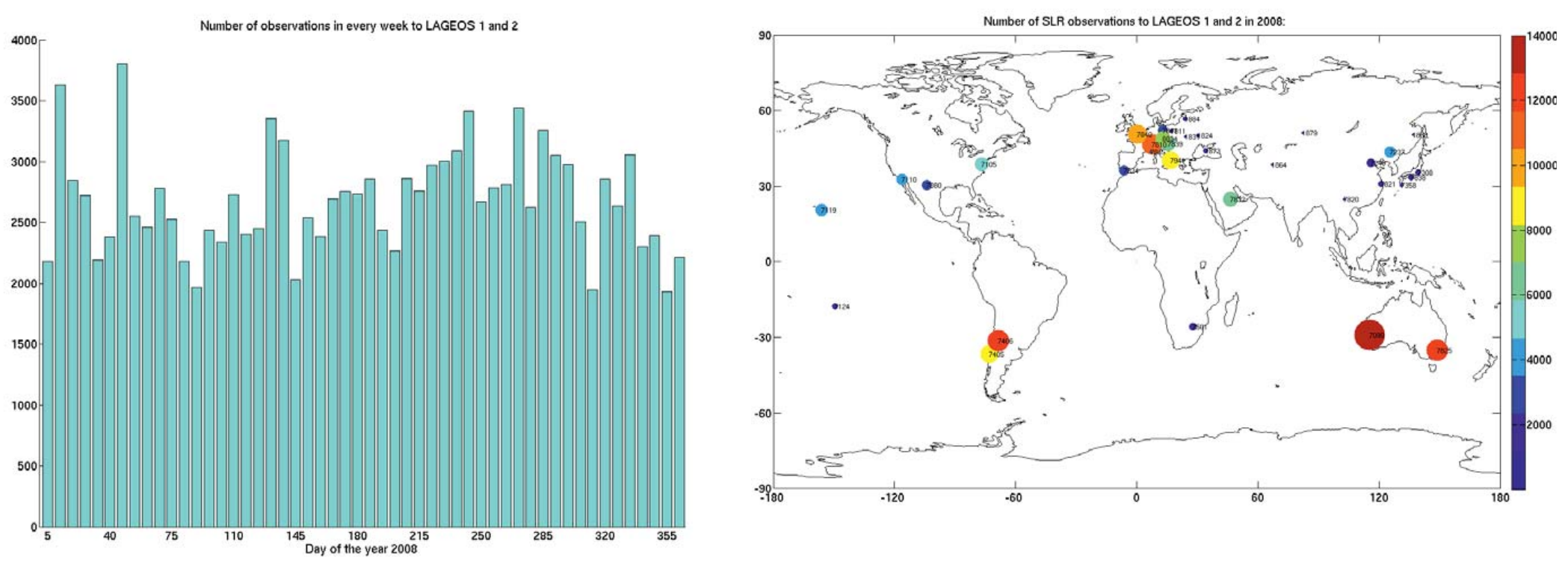

Fig. 1a,b: Number of SLR observations to LAGEOS-1 and LAGEOS-2 in 2008.

Maximum degree and order

We studied the impact of the maximum d/o on orbit determination of LAGEOS-1 and LAGEOS-2 when using the EGM2008 gravity field model. Figure 2 shows that the LAGEOS satellites are very sensitive up to d/o 14. Differences between the solutions based on degrees 12 and 14 are rather large. Small differences between degree 14 and 20 are still visible (on the level of about 0.5 mm). Increasing the maximum degree of the gravity field model beyond 20 has no significant impact on the resulting satellite orbits. The differences are at the level of 0.01 mm.

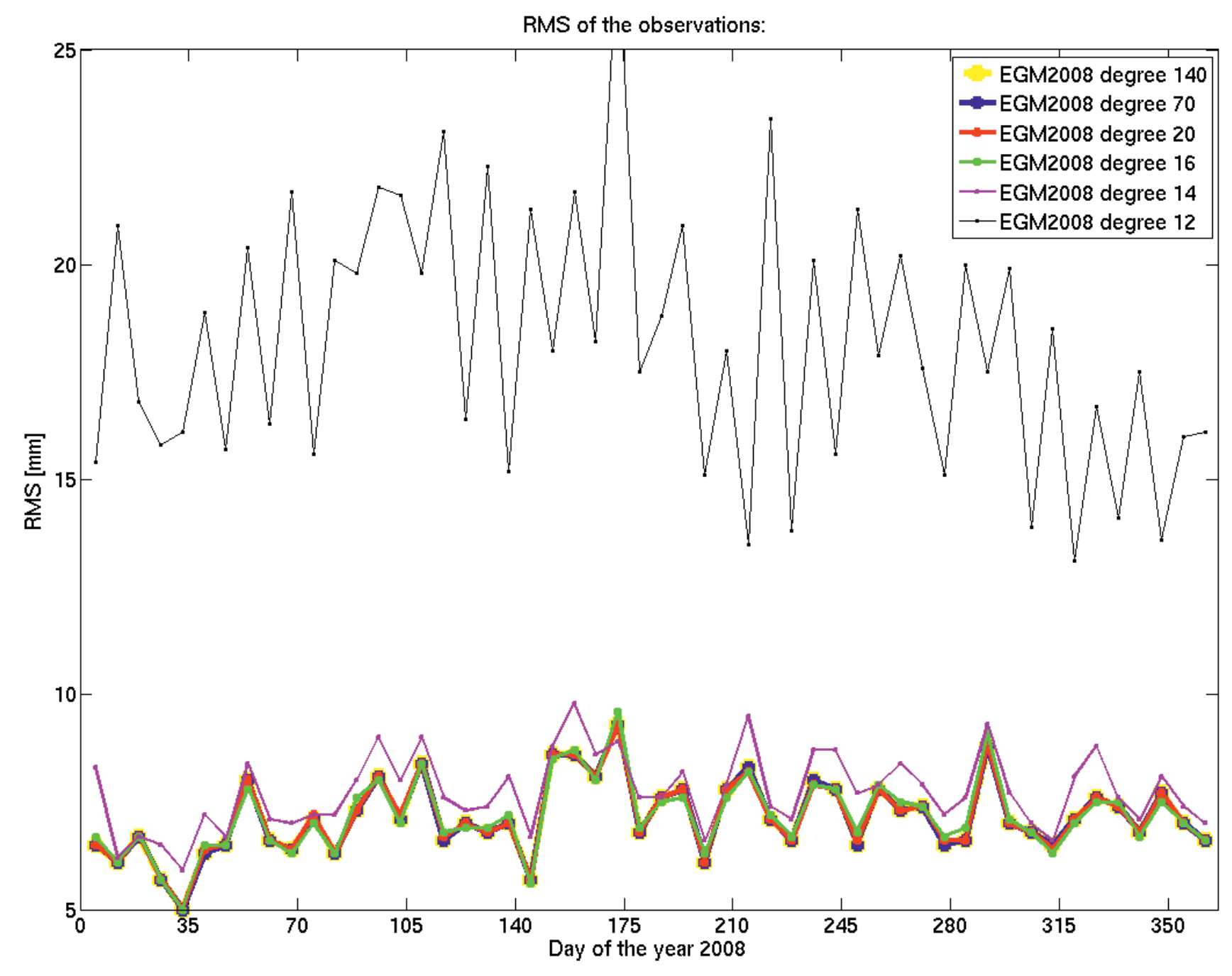

Fig. 2: RMS of weekly LAGEOS solutions obtained with the full set of empirical parameters (solutions (a)) for different maximum degrees of the EGM2008 gravity field model.

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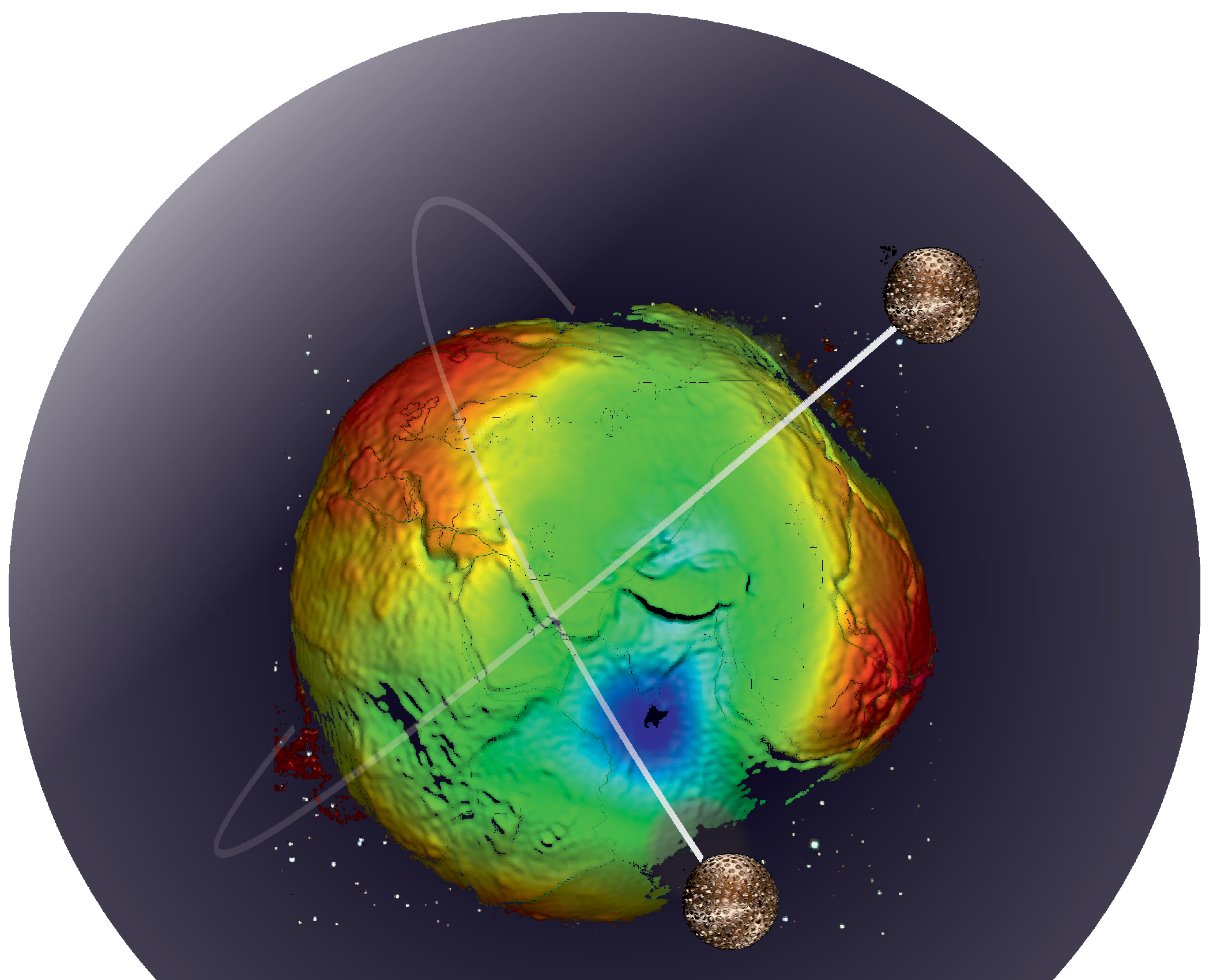


Comparison of gravity field models

The RMS of the observation residuals from the weekly solutions is an indicator of the quality of the underlying gravity field model. It is shown for solutions (a) and (b) in Fig. 3 (top) and (bottom), respectively.

Similar results are obtained for solutions (a), apart from the RMS values for JGM3 and ITG-GRACE2010S being slightly larger than for the other models. Smallest RMS values are obtained for EGM2008, AIUB-GRACE03S, EIGEN51C, EIGEN-GL04C (7.13, 7.15, 7.16, and 7.17 mm, respectively). The RMS of ITG-GRACE2010 could be reduced to 7.18 mm, provided that the coefficients C_{10}, C_{11}, and S_{11} are set to zero (as recommended). This modified model is further indicated as ITG-GRACE10 mod.

A more pronounced discrimination between the different models is obtained for solutions (b). Smallest RMS values are obtained for the GPS-only models AIUB-CHAMP03S and AIUB-SST-only, a rather large value for AIUB-GRACE03S.

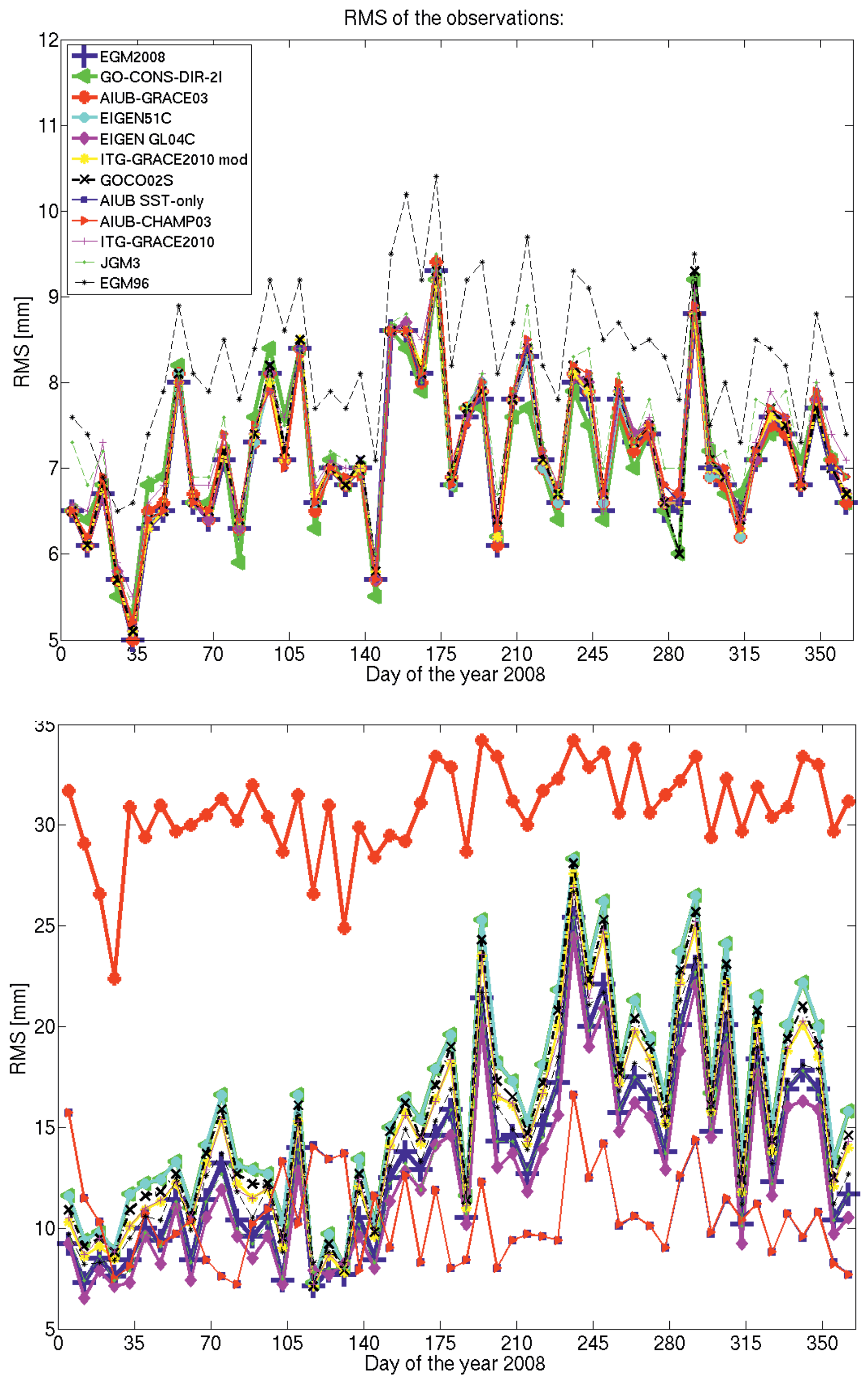
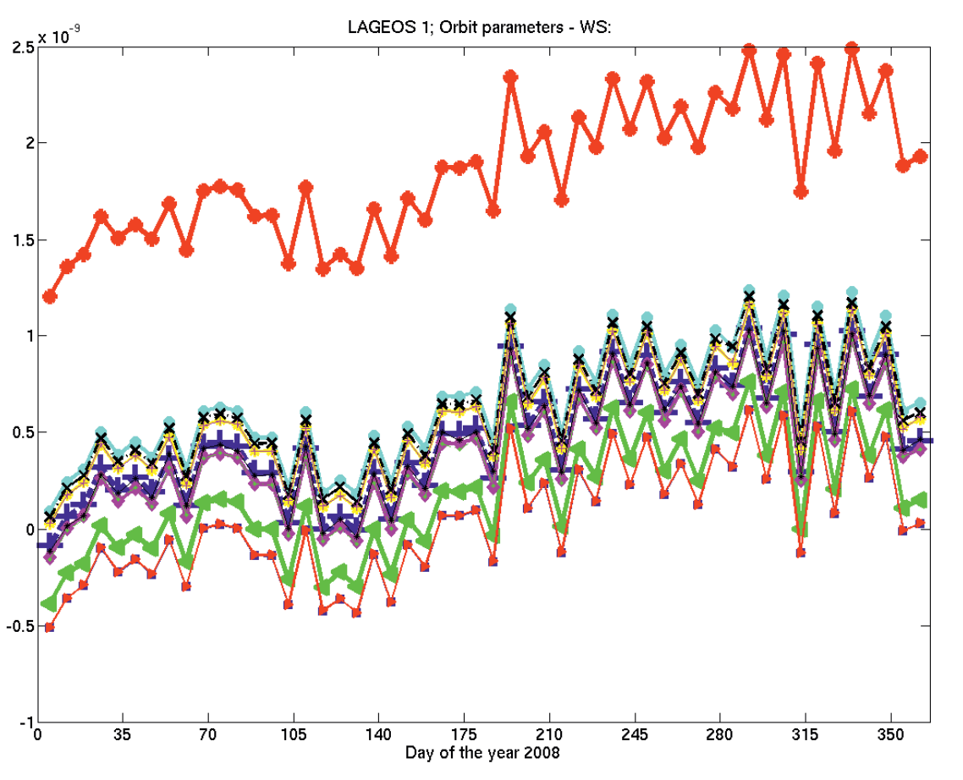


Fig. 3: RMS of weekly LAGEOS solutions with the full set of once-per-rev accelerations estimated (solutions (a), top) or without once-per-rev cross-track accelerations (solutions (b), bottom). Note the different scales.

Gravity field model	RMS of observation residuals [mm]	
	With estimation par. cross-track once-per-rev	Without estimation par. cross-track once-per-rev
EGM96	8.29	14.33
JGM3	7.42	13.28
ITG-GRACE2010S	7.32	15.05
AIUB-CHAMP03S	7.22	10.51
AIUB-SST-only	7.21	10.52
GOCO02S	7.20	15.55
ITG-GRACE10 mod	7.18	15.01
EIGEN-GL04C	7.17	12.56
EIGEN51C	7.16	16.19
AIUB-GRACE03S	7.15	30.74
GO-CONS-2-DIR-R2	7.14	16.20
EGM2008	7.13	13.40



Tab. 2, Fig. 4: Statistical values of Fig. 3, estimates of once-per-rev cross-track accelerations for LAGEOS-1 (sine-coefficient).

$$\begin{pmatrix} R' \\ S' \\ W' \end{pmatrix} = \frac{3}{2}\,\frac{\tilde{C}_{20}}{r^4} \begin{pmatrix} 1 - \frac{3}{2}\sin^2 i + \frac{3}{2}\sin^2 i \;\cos 2u \\ \sin^2 i \;\sin 2u \\ \sin 2i \;\sin u \end{pmatrix} \qquad \textbf{Eq. (1)}$$

Eq. 1: Perturbation acceleration due to C_{20} in the radial (R'), along-track (S'), and cross-track (W') directions as a function of the argument of latitude u. The full correlation between C_{20} and empirically estimated once-per-rev cross-track sine-coefficients is responsible that solutions (a) are almost insensitive to the quality of the C_{20} coefficient of the Earth's gravity field, e.g., yielding a good performance of AIUB-GRACE03S (Fig. 3, top), but large sine-coefficients (Fig. 4). Solutions (b) are well suited to directly reveal the quality of the C_{20} estimates, e.g., the reduced quality of C_{20} of AIUB-GRACE03S (Fig. 3, bottom).

Orbit comparison

Table 3 compares the solutions (a) based on different gravity field models. The RMS for the orbits based on ITG-GRACE2010 model is largest w.r.t. other orbits, but may be significantly reduced by setting C_{10}, C_{11} and S_{11} to zero (ITG-GRACE-10 mod). The model with the second largest differences to the other models is JGM3 with values larger than 6 mm. EGM2008 differs only slightly from other GRACE-based models and significantly from CHAMP-based models (see Fig. 5). Orbits using AIUB-GRACE03S, ITG-GRACE10 mod, EIGEN-GL04C and EIGEN51C models are of comparable quality (all GRACE-based). Smallest values for the comparison are achieved for the AIUB-CHAMP03S and the AIUB-SST-only models (the latter being the extension of the CHAMP-based model with GPS measurements from GOCE and thus almost identical for the low degrees).

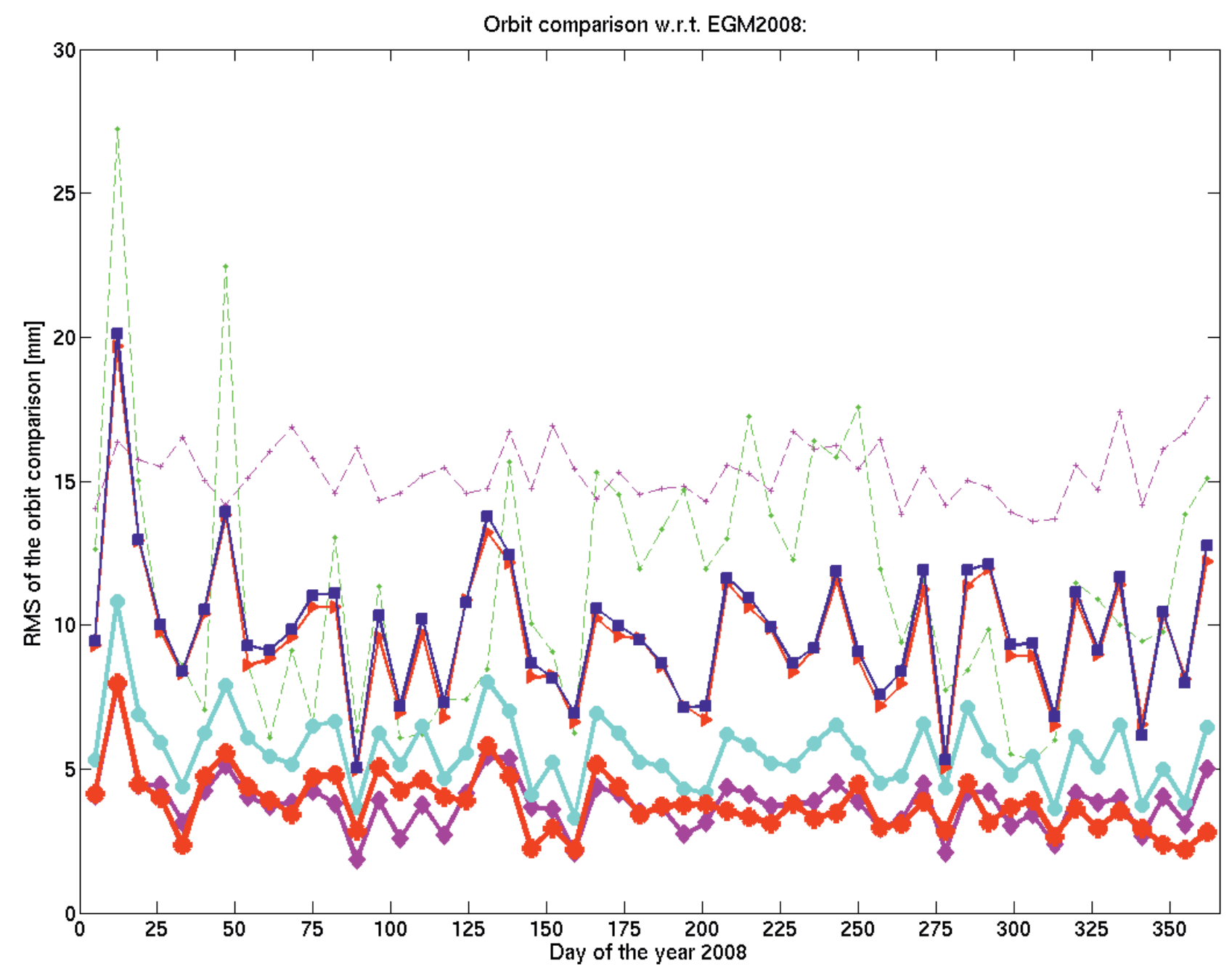


Fig. 5: Comparison between the orbits based on EGM2008 and other gravity field models. Same color code as in Fig. 3.

Gravity field model	EIGEN-GL04C	EGM2008	EIGEN51C	ITG-GRACE2010S	GOCE SST	AIUB-CHAMP03S	AIUB-GRACE03S	ITG-GRACE10 mod
JGM3	8.5	11.2	7.6	15.6	6.7	6.6	8.8	7.6
EIGEN-GL04C		3.8	2.1	14.3	6.2	5.9	1.8	1.2
EGM2008			5.6	15.3	9.8	9.5	3.7	4.3
EIGEN51C				14.1	4.4	4.2	2.4	1.9
ITG-GRACE2010S					14.7	14.6	14.5	14.1
GOCE SST						0.6	6.6	5.8
AIUB-CHAMP03S							6.4	5.5
AIUB-GRACE03S								2.2

Tab. 3: Comparison between estimated orbits based on different gravity field models: RMS of orbit differences (mean for 2008, in mm).

Estimation of C_{20}

Based on the parametrization of solutions (b), SLR data from 2009 to both LAGEOS satellites were used to estimate corrections to the C_{20} a priori values from the GGM02S gravity field model with the Bernese Software. Figure 6 shows the unconstrained weekly estimates as well as monthly estimates obtained by accumulating the weekly normal equations. The C_{20} series from CSR shows a fair agreement with the monthly estimates.

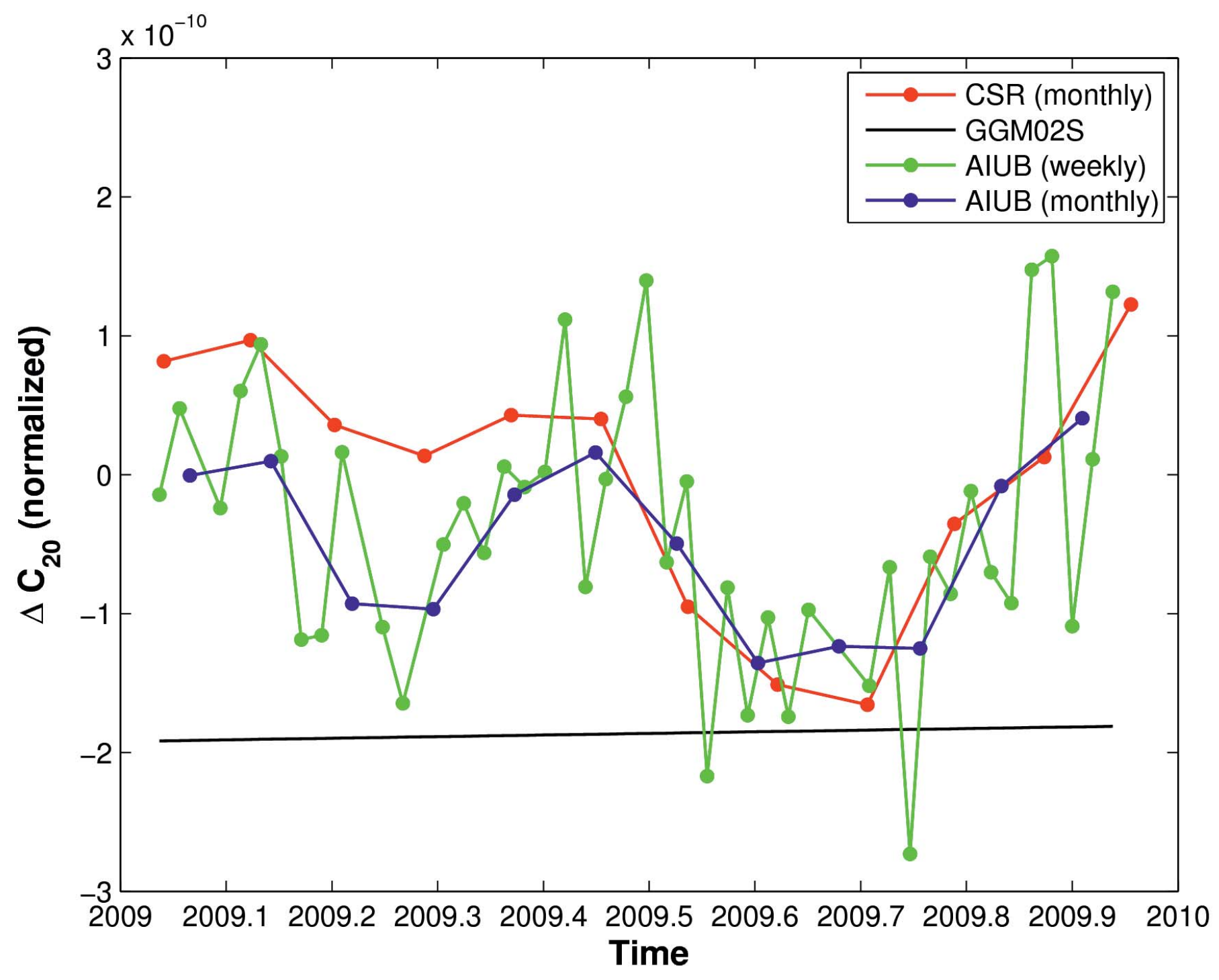


Fig. 6: Weekly and monthly estimates of C_{20} obtained with the Bernese GPS Software (AIUB). The CSR C_{20} series and the values from GGM02S are shown for comparison.

Summary

- The LAGEOS satellites are sensitive up to degree 20 of the underlying gravity field model.
- The smallest RMS of fit to the SLR data from LAGEOS-1 and LAGEOS-2 is observed for EGM2008, GO-CONS-2-DIR-R2, and AIUB-GRACE03S when estimating once-per-rev cross-track accelerations.
- Orbits based on JGM3 differ most w.r.t. orbits based on other models. The similar effect is observed for ITG-GRACE2010, when coefficients of degree one are not set to zero.
- The largest RMS of fit to the SLR data from LAGEOS-1 and LAGEOS-2 is obtained for AIUB-GRACE03S without estimating once-per-rev cross-track accelerations.
- First results of C_{20} estimates with the Bernese Software show a fair agreement with the C_{20} series from CSR. Longer data series will be processed in the near future.

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