

# An Estimation of the Number of Expected Returned Photons for the HartRAO Lunar Laser Ranger System

S. C. Ndlovu<sup>1,2</sup>, L. Combrinck<sup>1,2</sup>, P. Exertier<sup>3</sup>, M. Akombelwa<sup>2</sup> and N. Chetty<sup>2</sup>

1. Space Geodesy Programme, Hartebeesthoek Radio Astronomy Observatory, South Africa
2. College of Agriculture, Engineering and Science, University of KwaZulu-Natal, South Africa
3. French National Centre for Scientific Research, Observatoire de la Côte d'Azur, France

Corresponding Author: [sphume@hartrao.ac.za](mailto:sphume@hartrao.ac.za)

**Abstract:** Development of an integrated model and system to enable optimal efficiency and signal path parameter estimation of a Lunar Laser Ranger, is one of the major requirements for the new Hartebeesthoek Radio Astronomy Observatory's Satellite/Lunar Laser Ranger (S/LLR) system; without optimal efficiency of a lunar laser ranger signal path, the number of returned photons could be zero. The mathematical tool under development will be used to evaluate computed and observed photon return efficiency, using as departure point the existing link equation, with the option to add and estimate parameters in the least squares sense. The existing link equation can be used to predict the laser ranging system efficiency and is based on assumed accuracy of all parameters which influences the returned signal, presented as an estimate of expected number of returned photons. However, it does not make provision for model enhancements and parameter optimisations. Optimal efficiency in the S/LLR signal path will yield an improvement in the return-energy of the laser so that ranges to the lunar corner cube retro-reflectors can be measured accurately. This will ensure a high-precision measurement of the Earth-Moon distance, which is highly in demand since determination of the exact Earth-Moon distance is a complex undertaking. The geographic position of the HartRAO station, new state-of-the-art HartRAO S/LLR system under development and the expected number of returned photons will enable HartRAO to play a key role in improving the ranging accuracy to a sub-centimetre level, adding to the current effort to determine highly accurate Earth-Moon distances for various scientific purposes. We estimate the expected photon returns under various scenarios, including variable power levels, lunar distance, atmospheric conditions and system efficiency.

## Introduction

The HartRAO Lunar Laser Ranger (LLR) system requires a state-of-the-art software tool that enables optimal efficiency and signal path parameter estimation. Such a tool utilizes the existing link budget equation to estimate the number of returned photons for given conditions and LLR system parameters. It calculates the mean number of returned photons recorded by a photon detector as,

$$n_p = \eta_q \left( E_T \frac{\lambda}{hc} \right) \eta_t G_t \sigma \left( \frac{1}{4\pi R^2} \right)^2 A_r \eta_r T_a^2 T_c^2, \quad (1)$$

where all the parameters are as defined by Degnan, 1993.

In this work, we focus on the effects that result from the variable power loss, Lunar distance, atmospheric conditions and system efficiency. The link budget equation for the HartRAO LLR system is then written as,

$$n_p = C_s G_t \sigma \left( \frac{T_a T_c}{R^2} \right)^2, \quad C_s = \eta_q \left( E_T \frac{\lambda}{hc} \right) \eta_t \left( \frac{1}{4\pi} \right)^2 A_r \eta_r, \quad (2)$$

where  $C_s$  is the known constant of the "fixed" system parameters, the efficiency of the retro-reflector optical cross-section,  $\sigma$ , is still a complex and active study to investigate the degradation of returned photons from the reflectors (Murphy et al. 2010). The transmitter gain,  $G_t$ , equation is modified in such a way that it takes into account the obscuring secondary mirror's support structure (see Figures 1 and 2) that obscures the transceiver path expressed as,

$$G_t = \frac{4\pi}{\lambda^2} \left( \frac{2}{\alpha_t^2} \right) \left( e^{-\alpha_t^2} - e^{-\alpha_t^2 \gamma_t^2} \right) (A_t - A_{os}), \quad (3)$$

where  $A_{os}$  is the total area of the obscuring structures.

Development of an integrated model and system to enable optimal efficiency for the HartRAO state-of-the-art LLR signal path will yield an improvement in the return-energy of the laser, hence more data of high quality will be achieved from the only (currently) LLR station in the entire Southern Hemisphere (Combrinck and Botha 2013).

## Methodology

An advanced mathematical tool (Figure 3), utilising C++ code, which is still under development is used to estimate the expected number of returned photons for the HartRAO's LLR system. This tool allows the user to select the targeted satellite/reflector (see Figure 4) and adjust the varying parameters. It will be used to evaluate computed and observed photon return efficiency, using as departure point the existing link equation, with the option to add and estimate parameters in the least squares sense. The values for satellites/reflectors' cross section were calculated from (Degnan 2012) and the atmospheric transmittance and cirrus transmission were obtained from (Degnan 1993). The calculated transmitter gain for HartRAO's system is  $2.0 \times 10^{13}$ .



Figure 1: The 0.3 m mirror and spider mounted on the 1 m telescope.

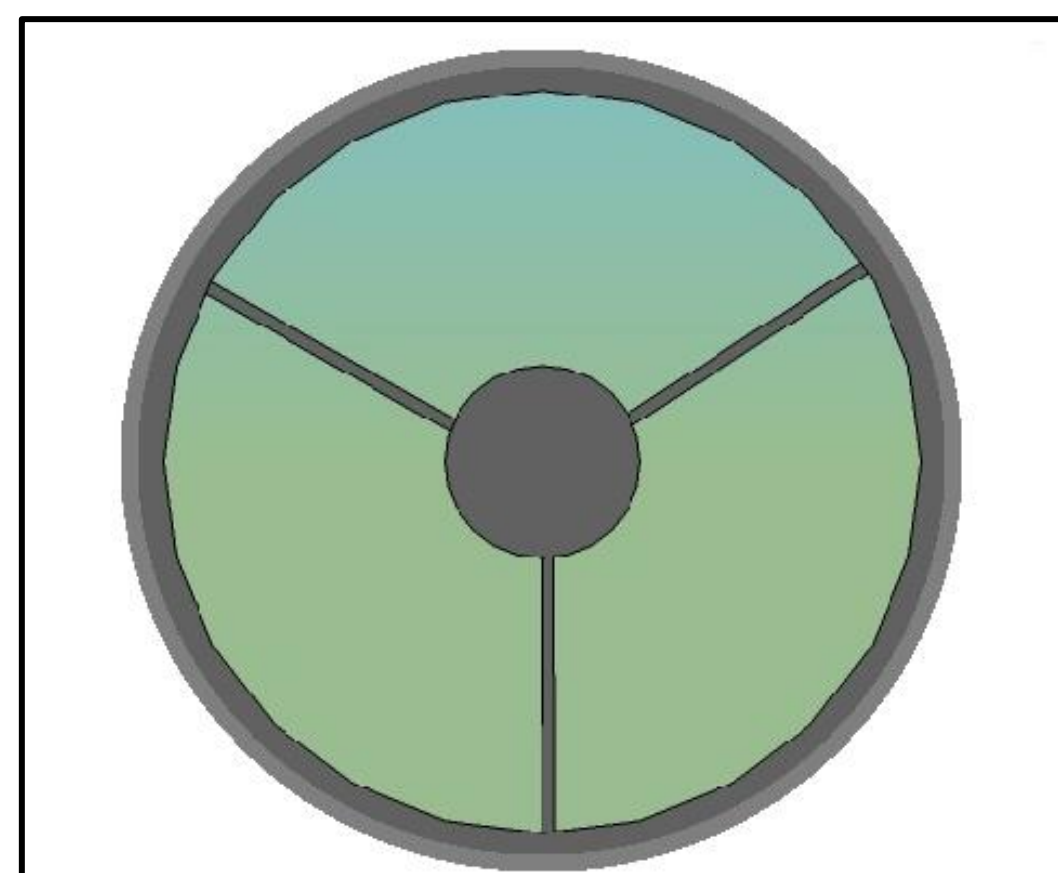


Figure 2: Schematic diagram illustrating the total surface area of the obscuring secondary mirror and spider.

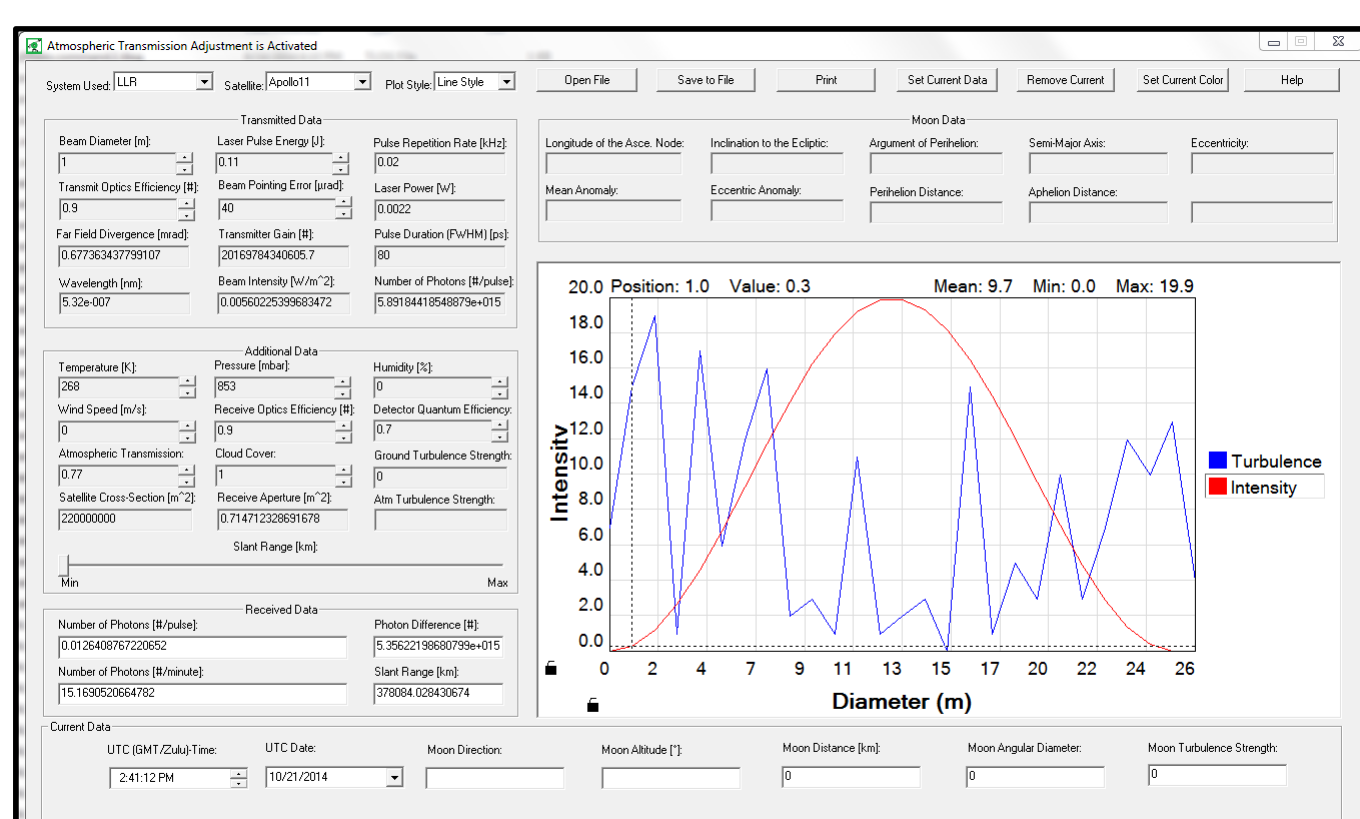


Figure 3: GUI of the under-development HartRAO program which estimates the received photons.

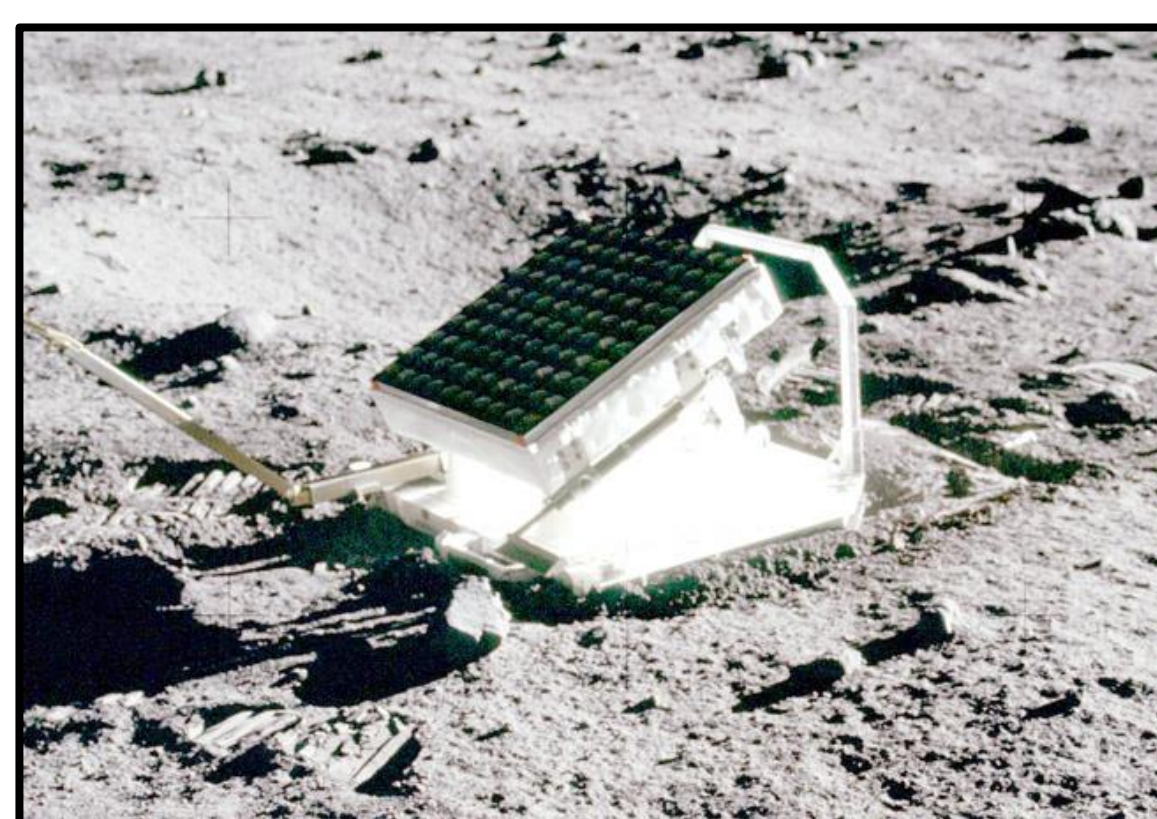


Figure 4: The Apollo 11 retro-reflector array placed on a dusty Lunar surface.

## Preliminary Results

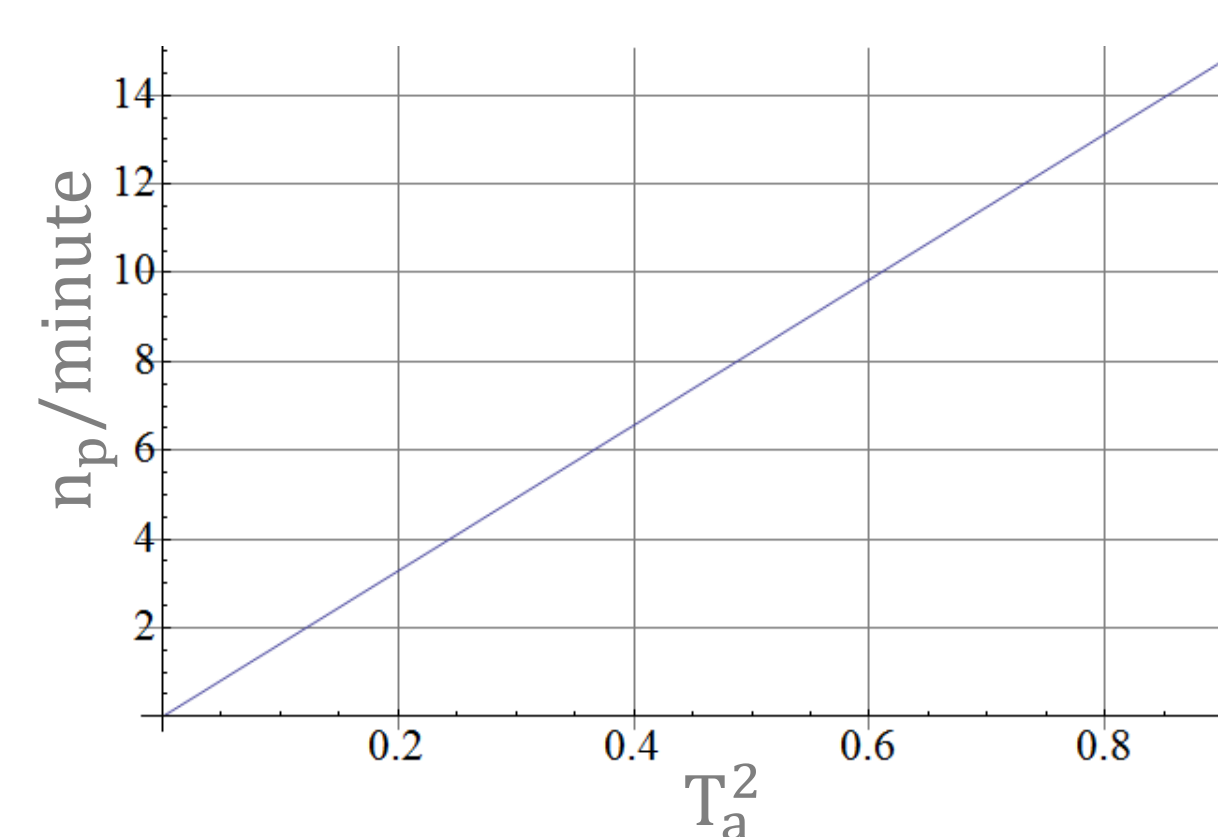


Figure 5: Received photons per minute vs. the varying atmospheric transmission.

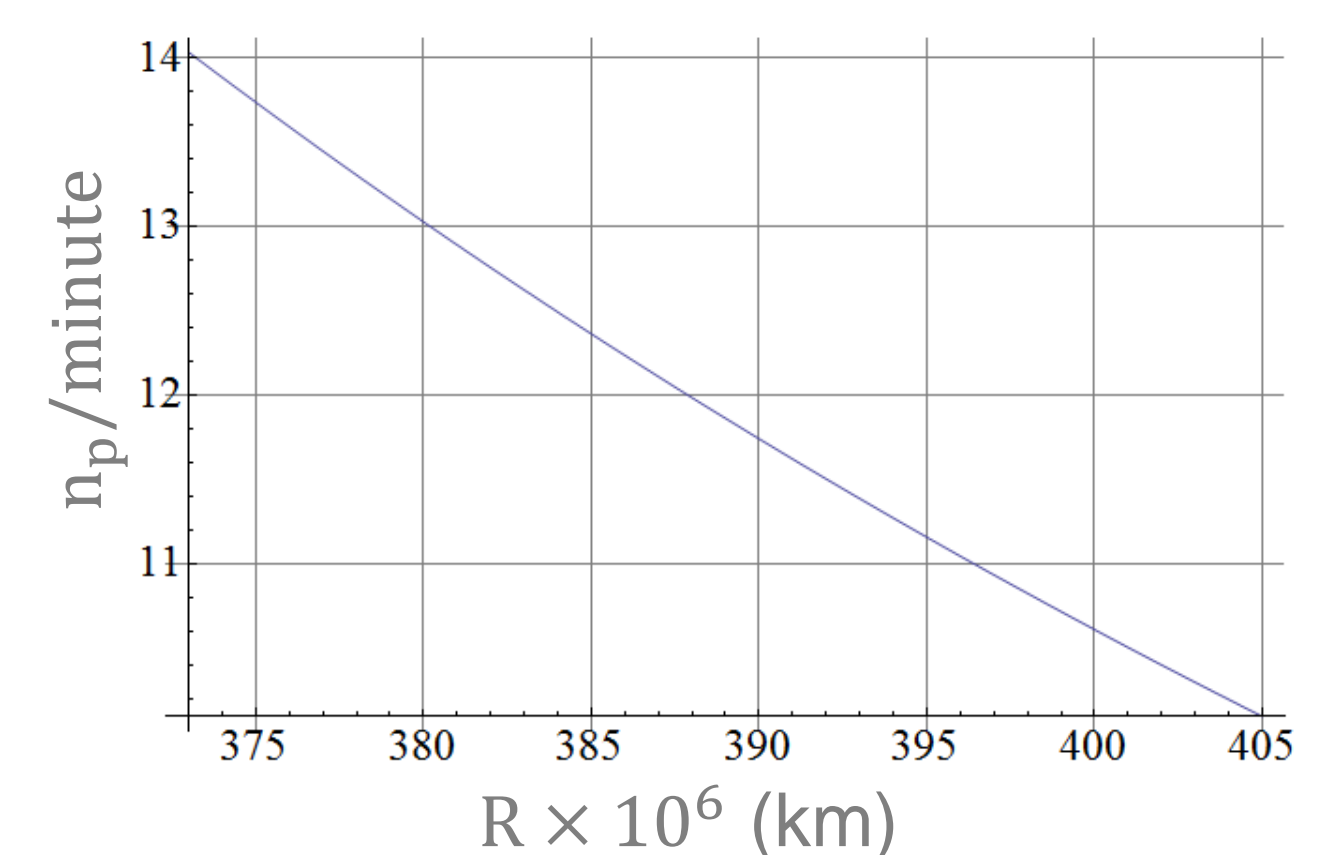


Figure 6: Received number of photons per minute vs. the slant range in  $10^6$  km.

Table 1: The relationship between varying parameters and number of photons reflected from Apollo 11 Laser Ranging Retro-Reflectors.

Parameter	Worst value	Optimal value
Laser pulse energy (mJ)	100	110
Transmit optics efficiency	0.4	0.9
Slant range (km)	405000	378084
Detector quantum eff.	0.4	0.7
Receive optics eff.	0.4	0.9
Atmospheric transmission	0.02	0.81
Cirrus transmission	0.1	1
Returned photons/minute	0.003	15

## Analysis and Discussion

The graphs (Figures 5 and 6) indicate the relationship between the atmospheric fluctuations, slant range and the expected number of returned photons. The slant range,  $R$ , is one of the biggest influences on the number of received photons (Figure 6). The maximum number of returned photons depends on "better" atmospheric conditions (Table 1) and this is in agreement with the available literature (Degnan 1993).

The mathematical tool will include those ranging degrading factors such as the angle of incidence, laser energy variations and efficiency of retro-reflectors, with the aim to add and estimate parameters in the least squares sense. The other restraining effects on the returned laser signal could result from thermal and density fluctuations of the atmosphere; this was recently investigated by our group (Ndlovu and Chetty 2014). Additional research is still necessary to verify the accuracy of the tool. This includes the use of the currently operating LLR stations' system parameters to evaluate computed and observed photon return efficiency.

## Conclusion

In conclusion, expansion of the existing link equation is a necessary requirement for a more accurate returned photon estimation. This will help in considering parameters (that affect ranging efficiency) and other factors, which can be caused by the detection system and obscuring system structures as the photons are transmitted and received through the LLR signal path. It will also improve the system signal-to-noise ratio, thus ensuring more photon returns for the HartRAO LLR system. Hence, more data can be achieved from the Southern Hemisphere station as the other existing LLR stations are all located in Northern Hemisphere.

## Acknowledgements

We thank the Department of Science and Technology (South Africa), Observatoire de la Côte d'Azur and NASA for their support.

## References

- Combrinck, L. and Botha, R. (2013), *Challenges and progress with the development of a Lunar Laser Ranger for South Africa*, 18<sup>th</sup> ILRS proc.
- Degnan J. J. (1993), *Millimeter accuracy Satellite Laser Ranging*, Geodynamics Series.
- Degnan J. J. (2012), *A tutorial on retroreflectors and arrays for SLR*, 17<sup>th</sup> ILRS proc.
- Murphy T. W., et al. (2010), *Long-term degradation of optical devices on the moon*, arXiv:1003.0713v1 [astro-ph.EP].
- Ndlovu S. C. and Chetty, N. (2014), *Analysis of the fluctuations of a laser beam due to thermal turbulence*, CEJP.