LOD systematics from SLR observations

M. Bloßfeld[1], H. Müller[1], U. Hugentobler[2], D. Angermann[1], M. Gerstl[1]
[1]Deutsches Geodätisches Forschungsinstitut (DGFI), Munich, Germany, blossfeld@dgfi.badw.de
[2]Technische Universität München (TUM), Munich, Germany

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Motivation

The excess Length Of Day (ΔLOD) describes the excess revolution time of the Earth w.r.t. 86400s. Laser ranging observations to satellites can be used in order to determine ΔLOD besides polar motion, crust-fixed station coordinates and coefficients of the Earth's gravitational field (Stokes coefficients). Reliable estimates of the previously mentioned parameters require the combination of different orbital inclinations since correlations between Earth rotation parameters, orbit parameters and the low degree Stokes coefficients affect the estimates. Furthermore, Satellite Laser Ranging (SLR) observations are sensitive to relativistic effects such as *Lense-Thirring* and *de Sitter* which are caused by the rotation of the Earth and the rotation of the Earth around the Sun. This paper discusses the existing correlations in theory and compares single-satellite solutions (LAGEOS 1, LAGEOS 2) with a two-satellite solution (LAGEOS 1/2) in order to quantify the secular effect of the parameter correlations on the estimated ΔLOD values.

Secular perturbations of LOD derived from satellite techniques

Based on Yoder et al. (1983) and Rothacher et al. (1999), the longitude of the ascending node Ω is highly correlated with the Earth's rotation around its z-axis (described by ΔUT1). This relationship can be expressed through

$$\frac{d}{dt}(\Delta UT1) = -\frac{\Delta LOD}{86400s} = -(\dot\Omega + \cos i \cdot \dot u_0)\rho^{-1} = -\left(\dot\Omega + \cos i \cdot (\dot\omega + \dot M_0)\right)\rho^{-1}.$$

Therein, ρ is the ratio of universal time to sidereal time, ω is the argument of perigee and M_0 is the mean anomaly at osculation epoch t_0. The parameters $\dot\omega$ and $\dot M_0$ are mapped with the cosine of the satellite's inclination i into the equatorial plane.

The dominating perturbation is caused by the even zonal Stokes coefficient C_{20} which describes the flattening of the Earth. As an example, the perturbation of Ω due to an offset of C_{20} using the first order Gaussian perturbation equation reads

$$\Delta\dot\Omega\Big|_{sec} = \frac{3}{2} a_\oplus^2 \sqrt{GM_\oplus}\,\frac{a^{-\frac{7}{2}}\Delta C_{20}}{(1-e^2)^2}\cos i\,.$$

Besides the secular perturbation due to ΔC_{20}, also the estimation of empirical once-per-revolution accelerations in cross-track direction (with s, c being the sine-, cosine-term) cause secular perturbations. The relationship is shown in the following equation:

$$\dot\Omega = \overbrace{\frac{rs}{2na^2\sqrt{1-e^2}\sin i}}^{\dot\Omega|_{sec}} + \overbrace{\frac{r(c\sin 2u - s\cos 2u)}{2na^2\sqrt{1-e^2}\sin i}}^{\dot\Omega|_{per}}.$$

Finally, relativistic effects such as the gravitomagnetic *(Lense-Thirring)* and the gravitoelectric precession *(de Sitter)* cause secular perturbations in ΔLOD according to

$$\dot\Omega_{LT}\Big|_{sec} = \frac{2GM_\oplus|J_\oplus|}{c^2a^3\sqrt{(1-e^2)^3}} \quad \text{and} \quad \dot\Omega_{ds} = \overbrace{\frac{3}{2}\frac{GM_\odot}{c^2|R|}n_\odot\sqrt{1-e_\odot^2}}^{\dot\Omega_{ds}|_{sec}}(1-\cos 2u).$$

The interaction of all perturbations is summarized in Figure 1. The impact on the mean velocity Δn is described in Bloßfeld et al. (2014).

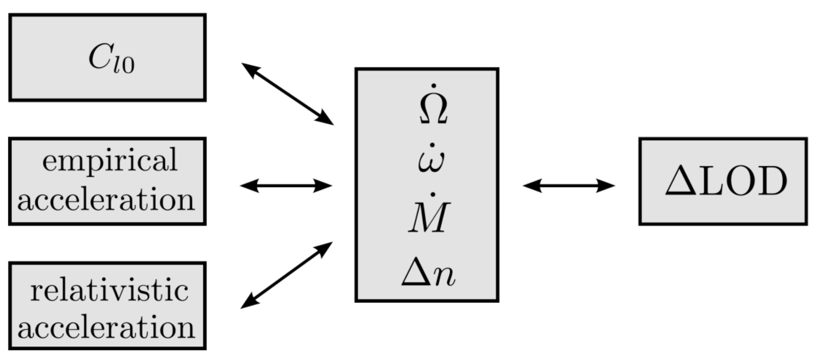


Fig. 1: Parameter relationships between the even zonal Stokes coefficients, the empirical and relativistic accelerations and ΔLOD.

DGFI SLR solution

The weekly DGFI single-satellite and multi-satellite SLR solutions contain station coordinates, Earth Orientation Parameters (EOP), orbit parameters and second degree Stokes coefficients. A detailed description of the estimated parameters and the solution setup (used constraints and parameterization) can be found in Bloßfeld et al. (2014). Figure 2 shows the correlation matrices of the single-satellite and multi-satellite solutions. Whereas the single-satellite solutions (upper middle and upper right panel) show high correlations near 1.0, both multi-satellite solutions (lower panels) show significantly decreased parameter correlations due to the mix of different inclinations.

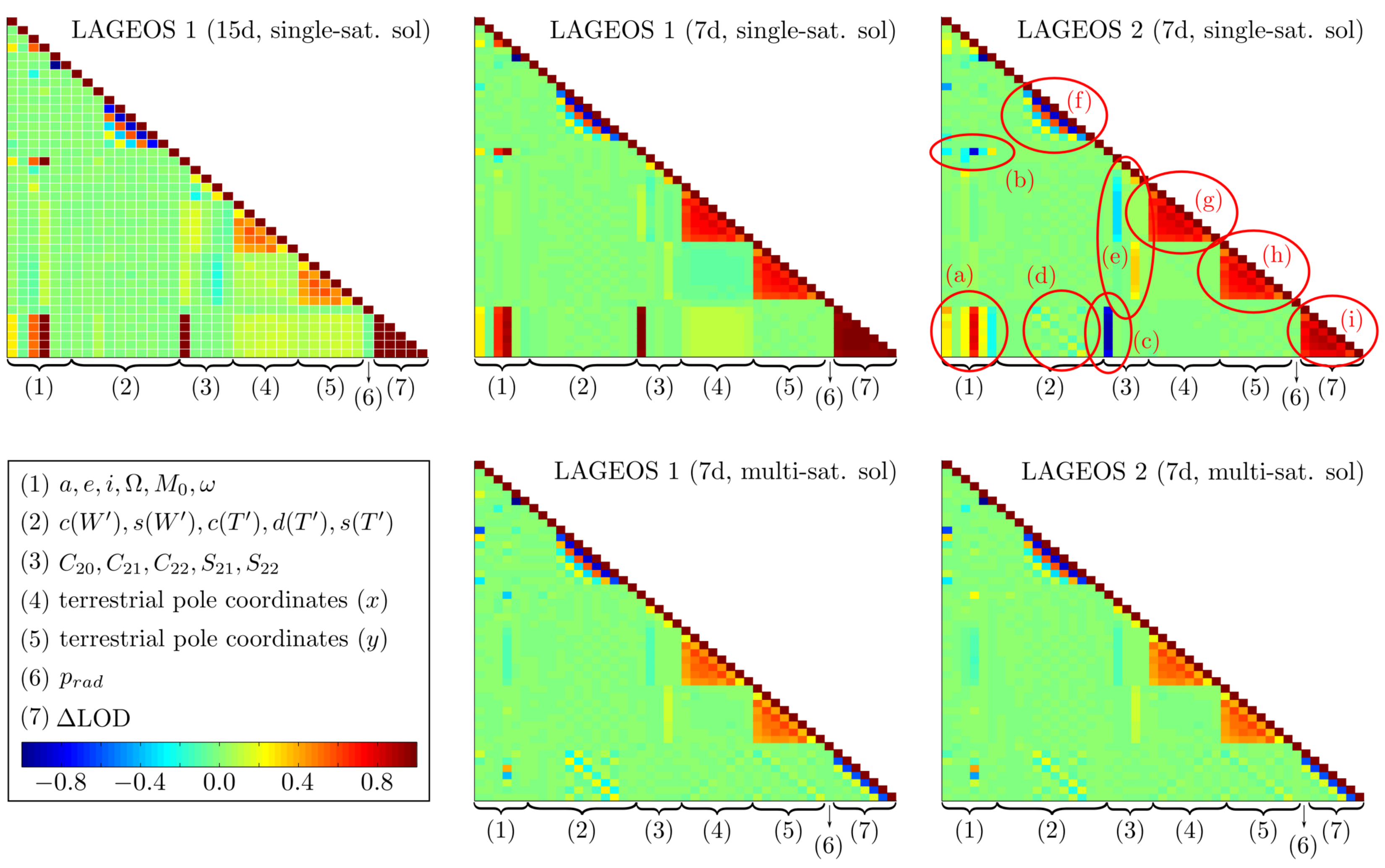


Fig. 2: Mean correlation matrices of LAGEOS 1 and 2 single-satellite and the satellite-separated multi-satellite solution. Due to the varying number of stations per week, the station-related (coordinates and biases) rows/columns are not shown. The correlations (a) to (i) are explained in Bloßfeld et al. (2014).

Systematics in LOD due to a priori gravitational fields

Since the ΔLOD estimates are highly correlated with C_{20}, different a priori gravitational fields result in different ΔLOD estimates. Figure 3 and 4 demonstrate the impact of four different C_{20} a priori values on ΔLOD. It can be clearly seen that the systematics are reduced in the multi-satellite solution.

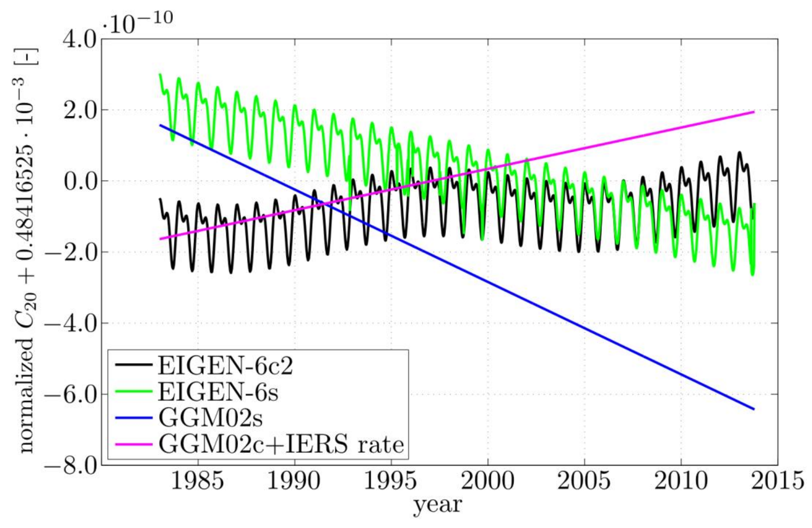


Fig. 3: Used a priori values for the Stokes coefficient C_{20} modeled over the whole computation period (1983.0 until 2014.0).

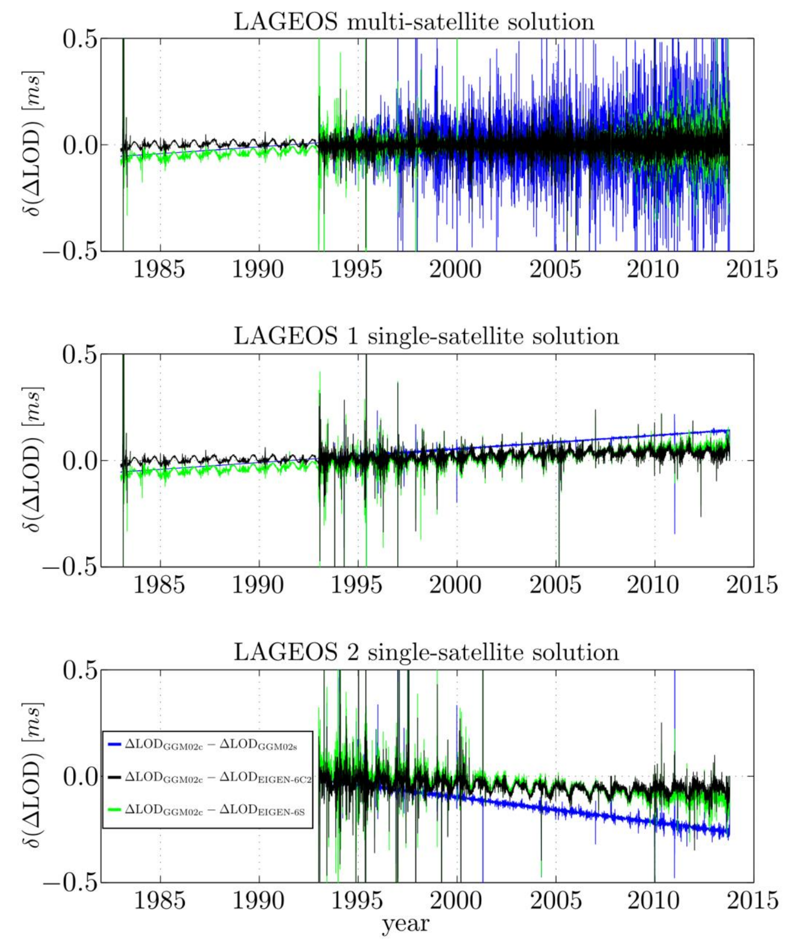


Fig. 4: Systematics of ΔLOD caused by different C_{20} a priori models.

Systematics in LOD due to orbit modeling and solution setup

In order to quantify the impact of the orbit modeling (estimation of empirical once-per-revolution terms) and the solution setup (estimation of Stokes coefficients), three different test solutions are computed:

sol 1: C/S_{2m} are fixed to GGM02C model and no empirical accelerations are estimated,
sol 2: C/S_{2m} are freely estimated and no empirical accelerations are estimated,
sol 3: C/S_{2m} and empirical accelerations are estimated (sine term s, cosine term c).

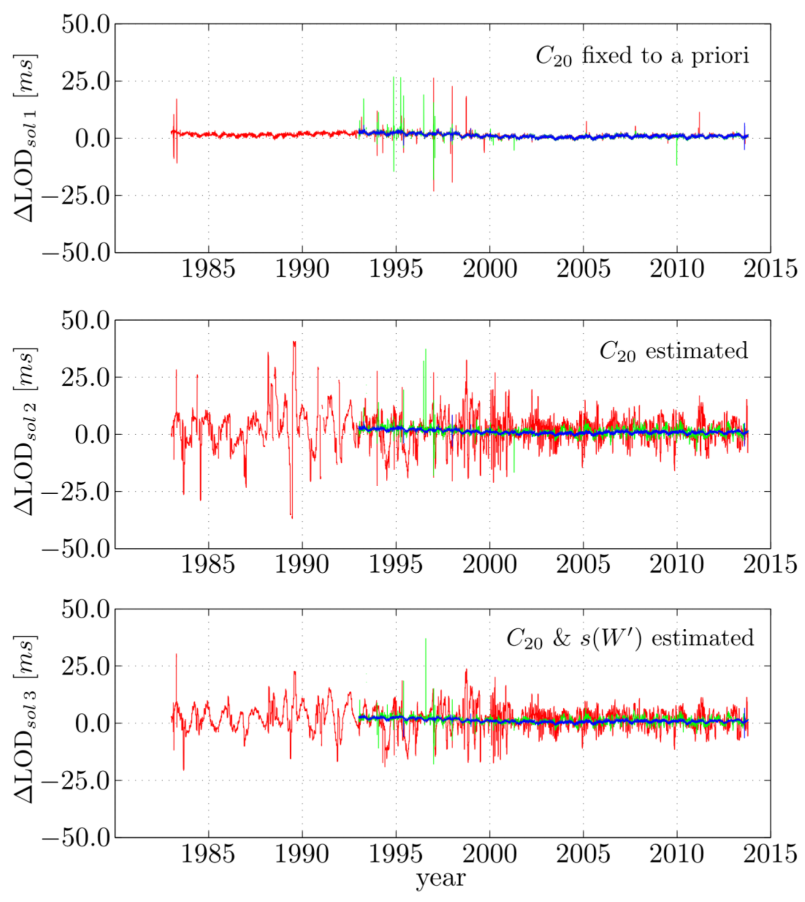
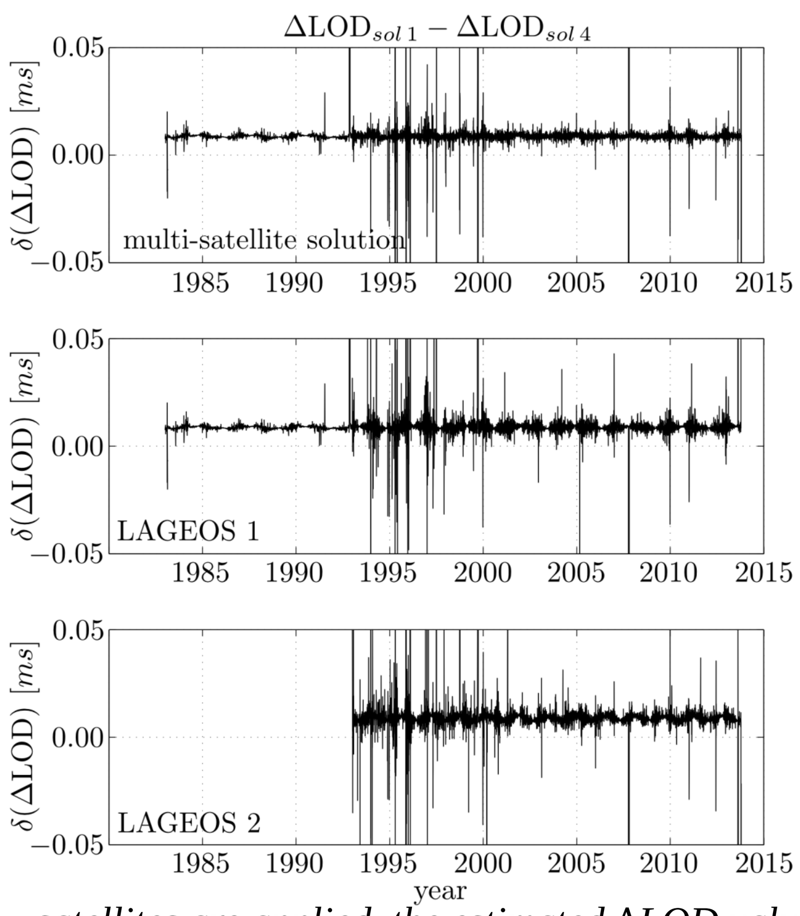


Figure 5 shows the ΔLOD estimates of the three different test solutions for both single-satellite and the multi-satellite solution. If C_{20} is fixed to a priori, the ΔLOD estimates depend on the a priori gravitational field (Figure 3 and upper panel in Figure 5). If C_{20} is freely estimated, no reliable ΔLOD estimates can be obtained in the adjustment in case of the LA1 solution. The LA2 estimates show only a small scatter which might be caused by the higher sensitivity of LA2 to C_{20} (due to the smaller inclination than LA1). The multi-satellite solution shows the smallest scatter. If also the s-term is estimated, the scatter of ΔLOD is reduced in all solutions but still no reliable estimates in case of LA1 can be obtained.

Fig. 5: Systematics of ΔLOD due to correlations with estimated parameters. The panels show the three LA1 (red) and LA2 (green) single-satellite solutions and the LA1/2 (blue) multi-satellite solution.

Systematics in LOD due to *Lense-Thirring* and *de Sitter*



The impact of the relativistic effects on ΔLOD can be obtained by comparing solution 1 with a forth test solution (see Figure 6):

sol 4: as sol 1 but no *Lense-Thirring* and *de Sitter* accelerations are applied.

The secular effect of both relativistic accelerations on ΔLOD is 8.7μs for both single-satellite and the multi-satellite solution. In addition, Figure 6 shows a periodic variation of the differences which might be caused by draconitic effects or variations in the Keplerian elements a and e (not investigated in this paper).

Fig. 6: Differences of ΔLOD values between sol 1 and sol 4. If no relativistic accelerations on the LAGEOS satellites are applied, the estimated ΔLOD values are systematically affected.

References & Acknowledgements:

Bloßfeld et al.: Systematic effects in LOD from SLR observations, Adv Space Res 54, pp: 1049-1063, doi: 10.1016/j.asr.2014.06.009, 2014
Rothacher et al.: Estimation of nutation using global positioning system, J Geophys Res 104(B3), pp: 4835-4859, doi: 10.1029/1998JB900078, 1999
Yoder et al.: Secular variation of Earth's gravitational harmonic J2 coefficient from Lageos and nontidal acceleration of Earth rotation, Nature 303(5920), pp: 757-762, 1983

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