Development of software for LLR data analysis at TianQin Research Center

W. Tian, H.-C. Yeh
TianQin Research Center for Gravitational Physics, School of Physics and Astronomy, Sun Yat-sen University, Zhuhai 519082, P. R. China. (tian-we@163.com)

Introduction

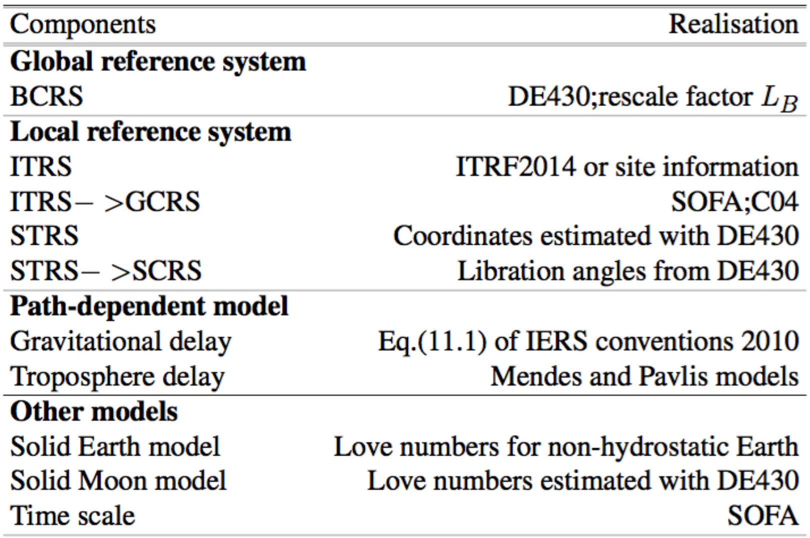
Lunar Laser Ranging (LLR) is an important tool for the investigations of the Earth-Moon system. In order to predict normal point data for new LLR stations and analyze observations from currently running LLR stations, a high precision LLR software package is being developed in our institute.

In the observation models, state vectors of major celestial bodies (including lunar orbit and libration) are obtained from the JPL's planetary ephemeris DE430; the temporal and spatial coordinates transformation is based on the IAU 2000 resolutions (Soffel et al., 2003); models related with Earth orientation, tide effects, gravitational and atmospheric time delay are computed on the basis of the IERS conventions (2010) (Petit et al., 2010). The prediction part of the package is finished. Comparison between calculated and observed (1-way) range from Apache Point, Grasse, Matera and MaDonald is presented.

Global celestial reference system used in the software

Since mean distance measured by Laser ranging to the Moon is much further than that to artificial satellites orbiting the Earth, the Barycentric Celestial Reference System (BCRS), rather than Geocentric Celestial Reference System (GCRS), is adopted in the analysis package to compute the range between LLR stations on the Earth and retroreflector arrays on the Moon. Orbital motion of main celestial bodies in solar system and libration of the Moon are calculated from JPL's DE430 (Folkner et al., 2014). Rescaling of spatial coordinates and mass parameters is needed due to transform of time scale from TDB adopted in the DE430 to TCB which is compatiable with BCRS.

Table 1: Reference systems and models in the work



Components	Realisation
Global reference system	
BCRS	DE430;rescale factor L_B
Local reference system	
ITRS	ITRF2014 or site information
ITRS$-$ >GCRS	SOFA;C04
STRS	Coordinates estimated with DE430
STRS$-$ >SCRS	Libration angles from DE430
Path-dependent model	
Gravitational delay	Eq.(11.1) of IERS conventions 2010
Troposphere delay	Mendes and Pavlis models
Other models	
Solid Earth model	Love numbers for non-hydrostatic Earth
Solid Moon model	Love numbers estimated with DE430
Time scale	SOFA

Local reference systems used in the software

Except the BCRS, several local reference systems are necessary to describe local events which occur during LLR experiment. Thus transformation between global and local reference system are involved in the computation (see Tab. 1).

For most of LLR stations, their coordinates and velocities are given in the International Terrestrial Reference Frame (ITRF), their coordinates are precisely known. For the Apache Point Observatory Lunar Laser-ranging Operation or APOLLO station, whose coordinates are not available in ITRF, its coordinates from the IMCCE's INPOP08 solution (Fienga et al., 2009) and from the estimation of Pavlov et al.(2016) are adopted separately.

Solid Earth tide effect on displacement of LLR stations is calculated up to the order of 4 along with love numbers for a non-hydrostatic Earth derived by Dehant et al (1999). Transformation between the International Terrestrial Reference System (ITRS) and GCRS is realized by a rotation matrix. The rotation matrix is calculated by related subroutines in the SOFA library (IAU SOFA Board, 2016) and Earth rotation parameters from IERS C04 series (Bizouard and Gambis, 2009).

The coordinates of five retroreflector arrays on the Moon (APOLLO 11, 14, 15 and Lunokhod 1, 2) in a frame based on principal axes and center of mass given in Williams et al.(2013) are adopted in our calculation. The computation of solid Moon tide effect on retroreflector arrays is similar to that of solid Earth tide, but the central body becomes the Moon. Lunar tidal love numbers given by Folkner et al.(2014) are used. Libration angles from DE430 is adopted to calculate the rotation matrix between the Selenocentric Terrestrial Reference System (STRF) and Selenocentric Celestial Reference System (SCRF).

Finally, the local celestial refernce frames GCRS for LLR stations on the Earth and SCRS for retroreflector arrays on the Moon are transformed to the global celestial reference frame BCRS on the basis of the IAU Resolution B1.3. The state vectors of celestial bodies used in transformation for spatial coordinates are from DE430.

Except the spatial coordinate transformation, transform of time scale is also critical to LLR data analysis. The time scale conversions in this work are mainly realized with the aid of the SOFA library.

Path-dependent Models

As the electromagnetic signals propagating through atmosphere, they experience delays which significantly affect the accuracy of laser ranging to the Moon, and must be modelled in the analysis software. The zenith delay model developed by Mendes and Pavlis (2004) and mapping function developed by Mendes et al. (2002) are adopted to calculate the time delay induced by atmosphere.

Gravitational delays induced by major celestial bodies are also important for LLR data analysis, for example, the Sun can give rise to several meters delay for one way range. Total gravitational delays caused by the Sun, the Earth, the Moon and other planets are calculated.

Results

To testify the models in the software, 21591 normal points from the Apache Point, Grasse, Matera and MaDonald are adopted as benchmarks. Fig.1 shows discrepancies between the observed and

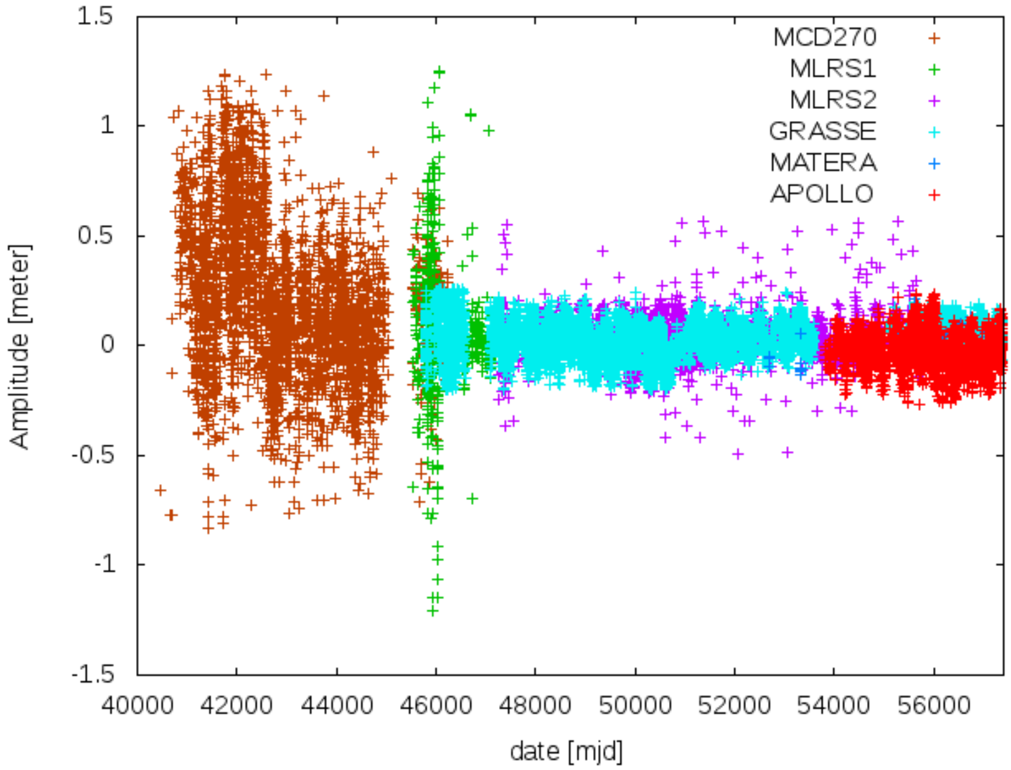


Figure 1. LLR measurement (one-way) residuals prior to adjustment of associated parameters. For a normal point, whose residual is larger than 3 times of *SD* of the corresponding LLR station, is eliminated from the figure. For the APOLLO station, estimated coordinates from INPOP08 are adopted.

calculated value (O-C) for 1-way range. The mean and standard deviation (*SD*) for each station is listed in Tab.2. Except McD270 (2.7m telescope) and MLRS1 (7086) at the McDonald observatory, standard deviations for other stations are less than 10cm. Even though statistic information given in Tab.2 do not represent the true measurement accuracy for each station, but these information demonstrates that models used in the package are qualified to predict the LLR normal point for currently running LLR stations and would be greatly helpful for us to finalize the whole LLR data analysis package.

Table 2. Statistic of O-C residuals for LLR stations. In last two columns both of AP and APa represent APOLLO station, but with station coordinates and velocities from two different solutions of INPOP08 and Pavlov2016 respectively. Ratio (*NP*1/*NP*0) represents the ratio between amounts of accepted normal points (3**SD* criteria) and total normal points for a station.

Station	McD270	MLRS1	MLRS2	Grasse	Matera	AP	APa
Date(yr)	69-85	83-88	88-15	84-15	03-15	06-15	06-15
NP_0	3604	631	3670	11180	118	2388	2388
NP_1	3485	566	3249	10826	104	2375	2369
Ratio	96.7%	89.7%	88.5%	96.8%	88.1%	99.5%	99.2%
Mean(cm)	21.0	7.1	3.2	2.8	-0.9	-2.7	2.9
SD(cm)	34.8	31.6	9.8	7.6	7.1	8.8	7.4

Since the O-C residuals in the work are calculated with fitted parameters from other solutions, they are impacted by the selection of fitted parameters, such as, coordinates of a LLR station. For instance, we calculate the O-C residuals for the Apache Point with estimated coordinates APOLLO(INPOP08) and APOLLO(Pavlov16) from two different solutions of Fienga et al.(2009) and Pavlov et al. (2016) respectively, the difference between them is shown in Fig.2.

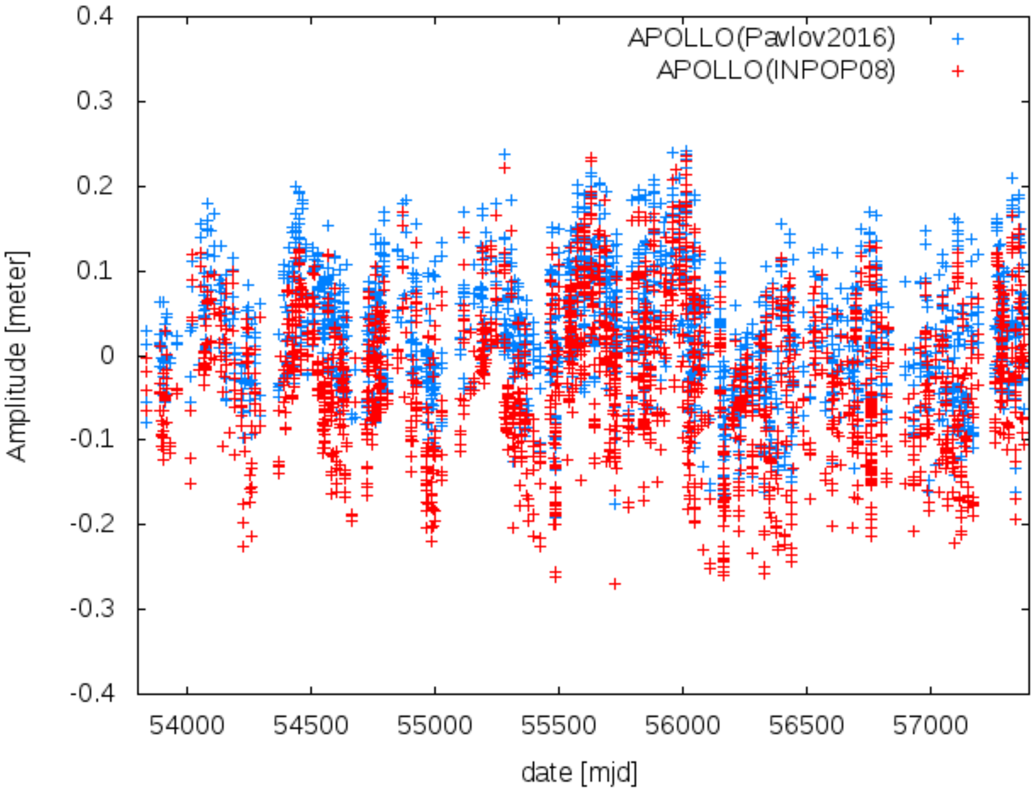


Figure 2. LLR measurement (one-way) O-C residuals for the Apache Point calculated with estimated coordinates from two different solutions.

Summary

In this presentation, models used in our LLR data analysis package are presented. the O-C residuals for normal points obtained from the Apache Point, Grasse, Matera and MaDonald in Fig.1 demonstrate that the prediction part of the package works effectively. However several correction models at the level of cm (models related with ocean tide loading, atmosphere loading, permanent tide, pole tide and frequency-dependent love number induced solid earth tide etc.) are lacking and will be supplemented in order to make the package as consistent as possible with the IERS and IAU conventions.

An independent high-precision LLR data analysis package should be able to fit parameters which are sensitive to LLR measurement, including dynamic parameters of Earth-Moon system. Thus, many modules are needed to be constructed in the near future and integrated in the final package. We are currently working on developing our dynamic model to fulfill the demands of millimetre LLR experiment.

Acknowledgements

Observations for the Apache Point, Grasse, Matera and MaDonald stations are downloaded from the Paris Observatory Lunar Analysis Center (POLAC) (http://polac.obspm.fr/llrdatae.html).

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