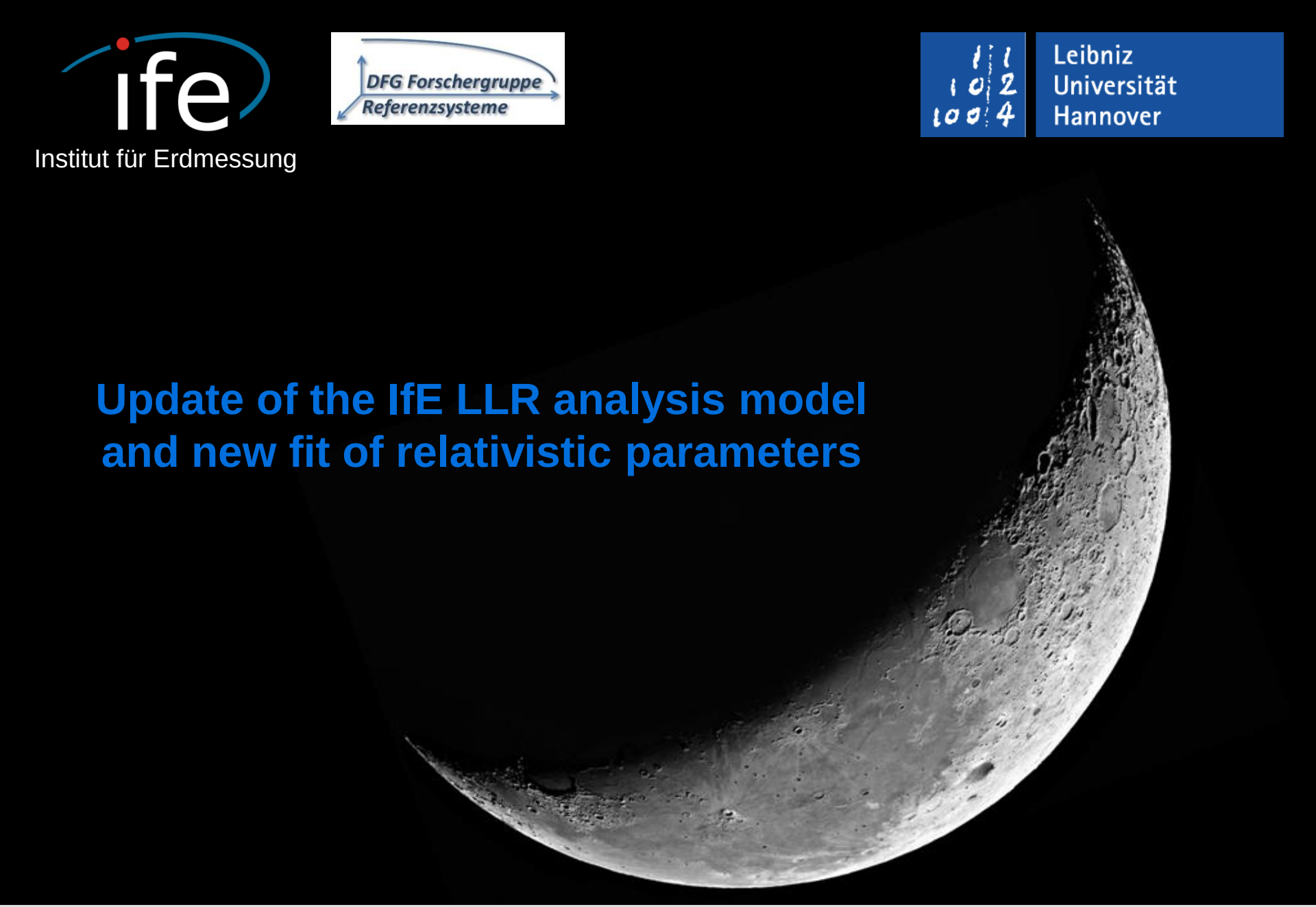
Institut für Erdmessung

DFG Forschergruppe Referenzsysteme

Leibniz Universität Hannover

Update of the IfE LLR analysis model and new fit of relativistic parameters



Franz Hofmann, Jürgen Müller, 11.10.2016

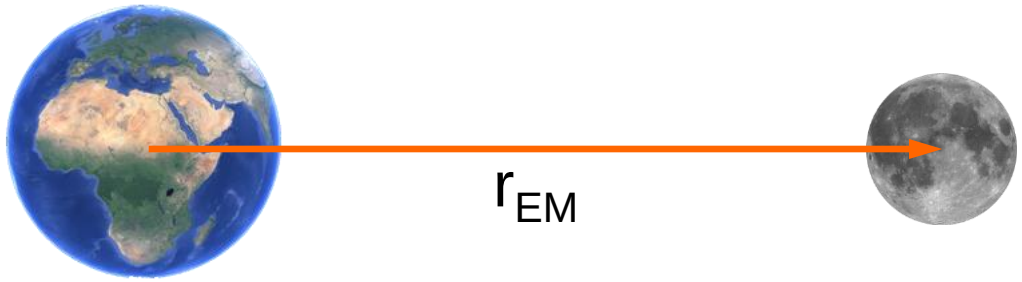
LLR analysis at IfE – program LUNAR

- Goes back to mid of 1980ies (FESG, Munich)
- 2 main development phases: ~1985-2001 (Munich), since 2006 at IfE

- LLR-only analysis
  - ephemeris computation based on initial values from DE421 (Sun, planets, largest asteroids)
  - no further planetary data (optical, radar,…) included
  - ephemeris model: EIH equations of motion for all bodies as point masses
  - Earth orientation: IERS conventions
  - lunar orientation: Euler equations integrated together with translation including relativistic corrections (geodetic precession, Lense-Thirring)
  - additional forces due to inhomogeneous gravity fields, tides
- Combined analysis of solar system data and LLR planned in future (project in research unit FOR 1503)

Program updates at IfE

- Data reduction compared with IMCCE, good agreement
- Results show some room for improvements → updates in ephemeris computation needed
  - many modeling parts still from first development phase
  - simplifications (due to computation time reasons and accuracy requirements)
  - slight inconsistencies in force model (‚interactions') resolved
  - 3 parts: Earth-related, additional gravitational effects, Moon-related



Program updates at IfE – Earth related

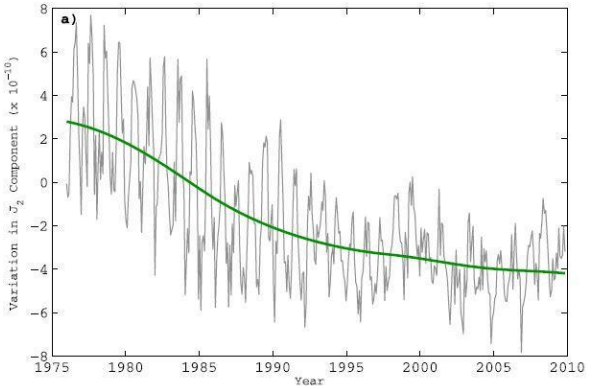
- Earth tides – tidal acceleration
  - former model
    - single lag angle of time delayed tides
    - only Moon as tide generating body
    - effect on lunar translation
  - **new model** (according to DE430 ephemeris)
    - degree 2 tidal potential (~98% of tidal effect)
    - arbitrary tide generating body possible
    - 5 tidal time delays (2 estimated, 3 fix at DE-values)
    - effect on Moon:
      - via change in Earth‘s degree 2 potential coefficients on lunar translation and rotation
      - via acceleration on lunar translation

Program updates at IfE – Earth related

- Earth tides – tidal acceleration
  - some **results**
    - Sun + Moon as tide generating body
    - tides from Jupiter, Venus → <0.1 mm in r_{EM} in 45 years
    - estimated time delays
      - diurnal τ_{21}=575 s (DE430: 636 s)
      - semi-diurnal τ_{22}=226 s (DE430: 219 s)

- Secular trend in C_{20}
  - several models tested (linear, quadratic)
  - best result in LLR analysis with model from IERS Conventions 2010 with

$$\dot{C}_{20} = 2.6 \times 10^{-11} \mathrm{yr}^{-1}$$



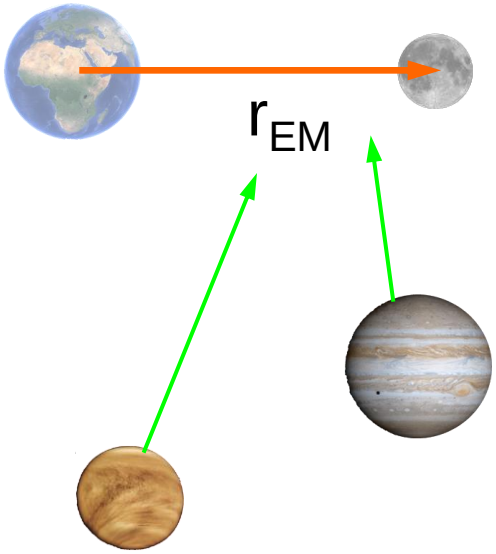
Roy and Peltier (2011)

Program updates at IfE – additional gravitational effects

- Interaction of Sun/planets with Moon
  - former model – interaction of
    - planets with point-mass Moon
    - Sun with lunar degree 2
  - **new model**
    - planets with lunar degree 2
    - Sun with lunar degree 2+3
  - **main effect** on lunar rotation, e.g.
    - ~ 19 mm on surface from Venus
    - ~ 4 mm on surface from Jupiter

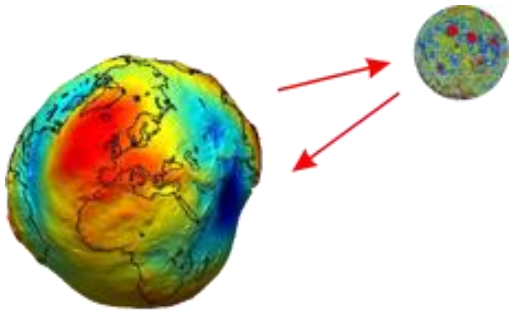
    (from ephemeris with equal initial conditions)
  - in analysis: residuals decrease ~0.1 mm on average

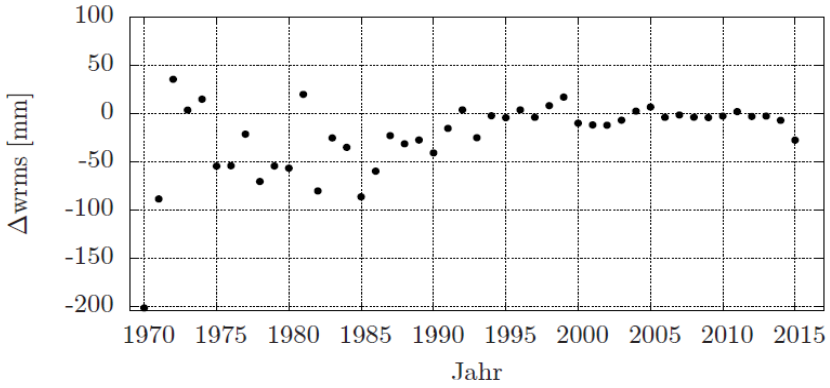
    → small effect but maybe needed in future



Program updates at IfE – additional gravitational effects

- Figure-figure interaction between Earth and Moon
  - former model
    - simplified version of degree 2 – degree 2 coupling
    - effect on lunar rotation considered
  - **new model**
    - coupling up to any degree/order of the gravitational field of Earth and Moon possible (Ilk, 1983)
    - effect on translation and rotation
  - **results**
    - improvement due to complete degree 2-degree 2 coupling
    - Earth degree 2 – Moon degree 3 → some mm on surface





Program updates at IfE – Moon related

- Rotation of deformable Moon
  - former model
    - tidally deformed tensor of inertia only in rotation
    - no changes of potential coefficients (interaction with Earth, additional effect on translation and rotation)
    - no consistent core-implemention
  - **new model** (according to 2-layer model of DE430)
    - basis tide-free tensor of ineria, elements from
      - C_{20} (GRAIL)
      - C_{22}, dynamical β (estimated in analysis)
      - dynamical γ derived
    - core moments with DE-fixed values for inertia-ratio C_{core}/C_{Moon} and core flattening

Program updates at IfE – Moon related

- tidal + spin deformations on mantle tensor of inertia
  - Love number k_2 fixed on GRAIL-value
  - tidal deformation due to Earth (much larger than from Sun)
  - 1 time delay used

→ equations for rotation complete

→ coupled differential equations for core + mantle

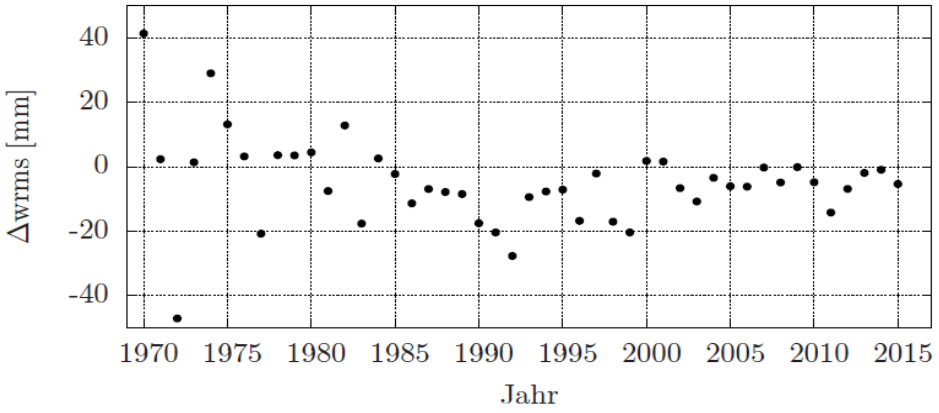
→ coupling constant, initial rotation vector of core estimated

- degree-2 changes in potential coefficients from deformed mantle tensor

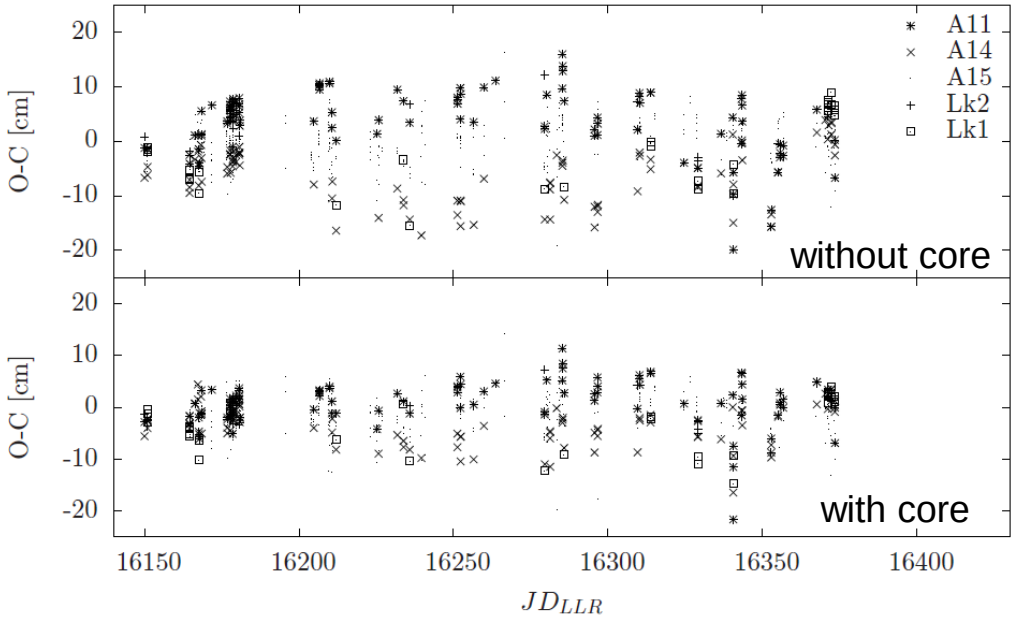
→ enters in computation of external forces (translation/rotation)

Program updates at IfE – Moon related

- Effect of new rotation-modelling on post-fit residuals, wrms (without core)

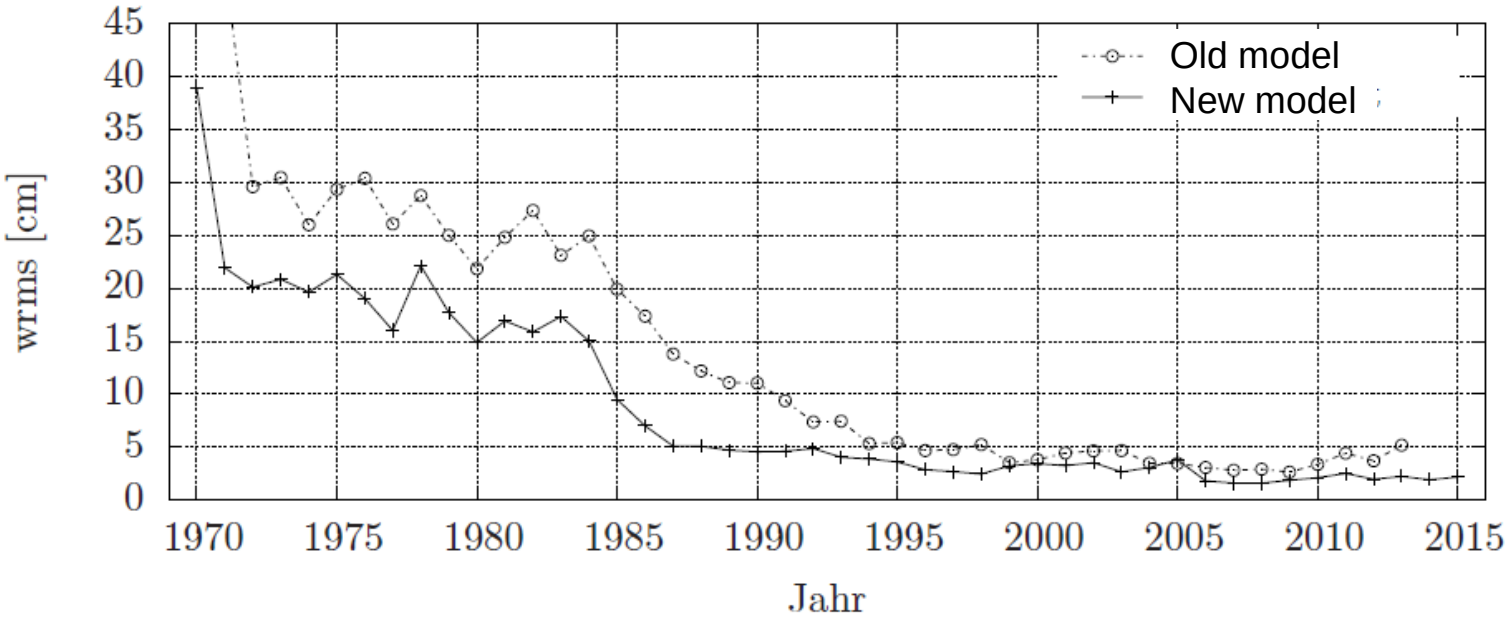


- Effect including core (which leads to improved libration modelling) on 2-way residuals



Results - wrms

- 1-way wrms, comparison with former model



- Further investigations
  - effects in longitude libration visible →empirical correction as in DE430
  - core flattening not yet estimated → possible effect on librations
  - tidal and lunar rotation modelling → room for improvement

Results – coefficients lunisolar nutation

Periode		MHB2000 [mas]	old model	new model (two test cases)	
18.6 a	A	-17206.42	1.89	0.17 ± 0.17	0.40 ± 0.17
	B	9205.23	-0.22	0.05 ± 0.06	0.02 ± 0.06
	A''	3.34	-3.71	0.67 ± 0.14	0.74 ± 0.14
	B''	1.54	-1.35	0.28 ± 0.06	0.28 ± 0.06
9.3 a	A	207.46	-0.88	-0.94 ± 0.11	-1.14 ± 0.12
	B	-89.75	-0.57	-0.55 ± 0.05	-0.65 ± 0.05
	A''	-0.07	-1.74	-1.04 ± 0.11	-1.25 ± 0.12
	B''	-0.03	-0.18	-0.07 ± 0.05	0.00 ± 0.05
365.3 d	A	147.59	-1.47	-0.29 ± 0.08	-0.50 ± 0.08
	B	7.39	0.58	-0.06 ± 0.03	-0.06 ± 0.04
	A''	1.12	-1.47	0.14 ± 0.06	0.20 ± 0.06
	B''	-0.19	-0.04	-0.01 ± 0.02	-0.04 ± 0.02
182.6 d	A	-1317.09	-1.49	0.49 ± 0.06	0.58 ± 0.07
	B	573.03	0.08	0.07 ± 0.02	0.12 ± 0.02
	A''	-1.37	1.22	-0.01 ± 0.06	-0.05 ± 0.06
	B''	-0.46	-0.08	0.04 ± 0.02	0.00 ± 0.03
13.6 d	A	-227.64	1.30	-	0.60 ± 0.24
	B	97.85	-0.69	-	-0.08 ± 0.10
	A''	0.28	-4.58	-	-0.83 ± 0.29
	B''	0.14	-2.54	-	-0.21 ± 0.11

less accurate, leads to larger discrepancies in other coefficients

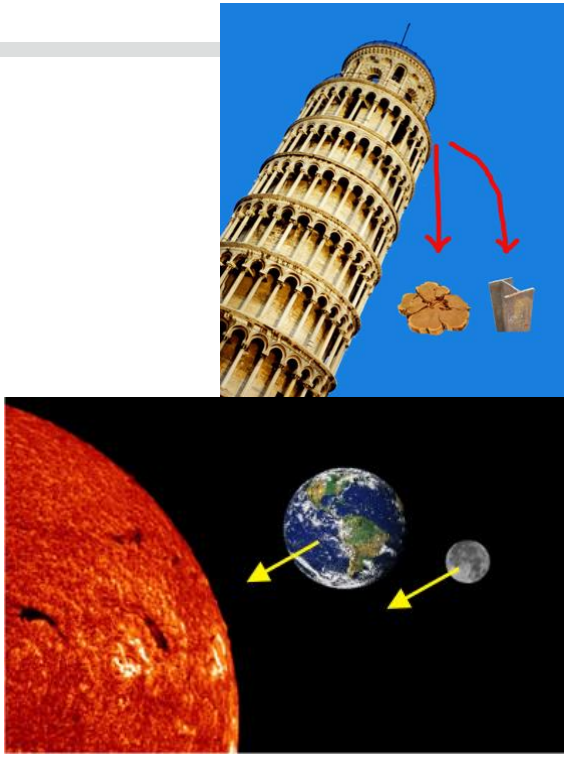
Results – relativistic parameters

- **Temporal variation of gravitational constant**
  - modelled as $G(t) = G_0 \left(1 + \frac{\dot{G}}{G_0} \Delta t + \frac{1}{2} \frac{\ddot{G}}{G_0} \Delta t^2 \right)$
  - moderate to strong correlations with
    - lunar core rotation vector (fixed in solution)
    - some station coordinates (constrained a little bit)
  - as single parameters: $\frac{\dot{G}}{G_0} = (0.7 \pm 0.8) \times 10^{-13} \mathrm{yr}^{-1}$

    $\frac{\ddot{G}}{G_0} = (1.6 \pm 2.0) \times 10^{-15} \mathrm{yr}^{-2}$
  - estimated together: $\frac{\dot{G}}{G_0} = (0.8 \pm 1.1) \times 10^{-13} \mathrm{yr}^{-1}$ $\frac{\ddot{G}}{G_0} = (-0.3 \pm 2.4) \times 10^{-15} \mathrm{yr}^{-2}$

Results – relativistic parameters

- **Equivalence principle**
  - estimating ratio $\Delta\left(m_g / m_i\right)_{EM}$
    - partials: computed numerically by introducing additional acceleration of Moon towards Sun
    - $\Delta\left(m_g / m_i\right)_{EM} = (-3.0 \pm 6.6) \times 10^{-14}$
  - Estimating Nordtvedt parameter η
    - partials: analytical from synodic range variation: 13.1m*cos(D)* η
    - $\eta = (-0.2 \pm 1.2) \times 10^{-4}$
  - into direction of galactic center (e.g. due to dark matter)
    - amplitude: $A_{gc} = 3.0 \pm 3.3\,\mathrm{mm}$
    - additional acceleration in parts of 1.9e-8 cm/s² $a_{gc} = (-1.1 \pm 1.2) \times 10^{-6}$



Results – relativistic parameters

- **PPN parameters β, γ**
  - included in EIH-equations of motion, partials numerically
  - correlated with station coordinates (constrained)
  - show also correlations with z-coordinate of lunar initial orbit values
  - $\beta - 1 = (0.9 \pm 1.0) \times 10^{-4}$
  - $\gamma - 1 = (-1.2 \pm 1.6) \times 10^{-4}$ (not as accurate as Cassini-result)
  - β from combination of PPN-parameters $0.25(\gamma+\eta+3)$ and Cassini- γ

    $\beta - 1 = (0.03 \pm 6.1) \times 10^{-5}$
- **PPN preferred frame α_1, α_2** w.r.t. cosmic microwave background
  - $\alpha_1 = (-1.1 \pm 2.0) \times 10^{-5}$
  - $\alpha_2 = (-0.6 \pm 0.9) \times 10^{-5}$ (not as accurate as test with Sun‘s spin)

Results – relativistic parameters

- **Geodetic precession** of lunar orbit
  - introducing GP a second time as additional acceleration
  - factor h gives relative deviation in from Einstein‘s theory (~1.9 as/cy)
  - strong correlation with
    - lunar core rotation vector (fixed)
    - z-component of lunar initial velocity (fixed)

$$h = (-0.6 \pm 2.0) \times 10^{-3}$$

- **Yukawa-term** (1/r²-test), acceleration due to $V_{EM} = -\frac{GM_E M_M}{r}\left(1 + \alpha e^{-r/\lambda}\right)$
  - interacting range λ = 380000km
  - coupling constant α estimated
  - correlations and fixed values like GP
  - $\alpha = (-4.0 \pm 5.0) \times 10^{-12}$

Summary

- IfE-LLR ephemeris model updated
  - tidal acceleration, secular trend in Earth‘s C_{20}
  - additional gravitational interactions planets-Moon, Earth-Moon
  - lunar rotation as 2-layer core/mantle model
  - effect of lunar deformation in all lunar equations of motion (translation plus rotation)

- Smaller residuals and more accurate parameter estimation
  - increased accuracy in relativistic parameters (strong limits for validity of equivalence principle and gravitational constant)
  - no deviation from Einstein‘s theory of gravity up to now